2D ELASTIC WAVEFORM INVERSION IN THE FREQUENCY DOMAIN

(Thesis format: Monograph)

by

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Graduate Program in Geophysics

A thesis submitted in partial fulfillment
of the requirements for the degree of
Master of Science

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The University of Western Ontario
London, Ontario, Canada

THE UNIVERSITY OF WESTERN ONTARIO

School of Graduate and Postdoctoral Studies

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entitled:

2D Elastic Waveform Inversion in the Frequency Domain

is accepted in partial fulfillment of the
requirements for the degree of
Master of Science

.....
Date

.....
Chair of the Thesis Examination Board

Abstract

150 words for MSc; 350 words for PhD.

Scrutiny of the Abstract

by Kenneth K. Landes (Editor of the AAPG Bulletin, Ann Arbor, Michigan)

A B S T R A C T

The behavior of editors is discussed. What should be covered by an abstract is considered. The importance of the abstract is described. Dictionary definitions of "abstract" are quoted. At the conclusion a revised abstract is presented.

Presumably new editors, like new senators and small children, should be seen and not heard. But unfortunately the Association has elected (the electorate had no choice) an editor who is a nonconformist. For many years I have fretted over the inadequate abstract, and now perhaps I can do something about it--but not by keeping quiet.

Many of the abstracts appearing in the publications, Including the meeting programs, of the A.A.P.G. can best be described by the use of a homely word that refers to an infestation by a certain minute organism. The abstract appearing at the beginning of this note is in that category. I regret to say that it is not an extreme case. My collection contains several that are worse. Dean Russell of Louisiana State refers to such abstracts as "expanded titles." They could also be looked upon as a table of contents, in paragraph form, with "is discussed" and "is described" added so as to furnish each subject with the verb necessary to complete the sentence. The reader is left completely in the dark not as to what the paper is about but as to what it tells! The information and the interpretation contained therein remain a mystery unless the reader takes the time to read or listen to the entire paper. Such abstracts can be likened to the "teasers" which your local movie manager shows you one week in the hope of bringing you back next week. But the busy geologist is more likely to be vexed than intrigued by the coy abstract.

To many geologists, especially the tyros in exposition, the writing of an abstract is an unwanted chore required at the last minute by a rule-ridden editor or insisted upon even before the paper has

been written by a deadline-bedeviled program chairman. However, in terms of the market reached, the abstract is the most important part of the paper. For every individual who reads or listens to your entire paper, from ten to five hundred will read the abstract. It is much better to please than to antagonize this great audience. Papers written for oral presentation should be prepared with the deadline the abstract date instead of the delivery date. Later discoveries can be incorporated within the paper--and they would miss the program abstract anyway.

My dictionary describes an abstract as "a summary of a statement, document, speech, etc." and "that which concentrates in itself the essential qualities of anything more extensive or more general, or of several things; essence." The definition I like best has been set in italics. May all writers learn the art (it is not easy) of preparing an abstract containing the essential qualities of their compositions! With this goal in mind I append an abstract that I believe to be an improvement over the one appearing at the beginning of this discussion.

A B S T R A C T

The abstract is of utmost importance for it is read by 10 to 500 times more people than hear or read the entire article. It should not be a mere recital of the subjects covered, replete with such expressions as "is discussed" and "is described." It should be a condensation and concentration of the essential qualities of the paper.

Keywords: Anisotropy, Elastic, Frequency domain, Near-surface, Oil and gas, Seismic processing, Shear wave, Utica shale, Velocity model-building, Waveform inversion

Statement of Co-Authorship

Coauthorship statement here.

Epigraph

An eipgraph is “a short quotation or saying at the beginning of a book or chapter, intended to suggest its theme.”

> For everything there is a season, and a time for every matter under heaven:
> a time to be born, and a time to die;
> a time to plant, and a time to pluck up what is planted;
> a time to kill, and a time to heal;
> a time to break down, and a time to build up;
> a time to weep, and a time to laugh; a time to mourn, and a time to dance;
> a time to cast away stones, and a time to gather stones together;
> a time to embrace, and a time to refrain from embracing;
> a time to seek, and a time to lose;
> a time to keep, and a time to cast away;
> a time to tear, and a time to sew;
> a time to keep silence, and a time to speak;
> a time to love, and a time to hate;
> a time for war, and a time for peace.
> - Ecclesiastes 3:1-8

Dedication

A dedication is “a message at the beginning of a book, song, etc., saying that it was written or is being performed in order to honor or express affection for someone.” I dedicate ...

Acknowledgements

First, I thank ...

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List of Abbreviations, Symbols, Nomenclature

σ_{ij}	Stress
u_x	Displacement in the x-direction

Preface

A preface is an “introduction to a book, typically stating its subject, scope, or aims.”

Chapter 1

Introduction - 2016 Western Thesis Regulations

The following details the regulations surrounding the process of producing, submitting, examining and publishing graduate theses at Western. For an overview of processes, services and support relating to theses please visit the Current Students section "Thesis" for more information.

1.1 GENERAL THESIS REGULATIONS

A thesis (or dissertation) is a formal statement of the theory, source materials, methodology, and findings of a student's major research project. It must be a complete and sufficient document that does not require subsidiary information to substantiate its findings. The examination of the thesis exposes the student's work to scholarly criticism.

To fulfill the degree requirement, the thesis and the candidate's oral examination must be assessed and approved by a Thesis Examination Board and meet SGPS requirements for thesis form and thesis content.

For each student writing a thesis, programs are required to establish a formal Thesis Advisory Committee for all thesis-based Masters and PhD students consisting of a supervisor and at least one other person. For information on SGPS Membership, please consult SGPS Membership.

1.1.1 Doctoral Programs

Every candidate for the Doctoral degree must complete a thesis. The thesis must indicate in what respects the investigation has increased knowledge of the subject. A candidate may not submit a thesis that has been previously accepted for a degree, but may, with the permission of the Graduate Program, incorporate material included in a previous thesis.

Doctoral candidates must present a Public Lecture on their thesis research. (SGPS has allowed certain programs to waive this requirement.) The Public Lecture allows the candidate to present his/her research projects to a UWO community of scholars in an open forum.

1.1.2 Master's Programs

Programs may require a thesis or make a thesis optional.

1.2 COMPLETION OF ALL NON-THESIS DEGREE REQUIREMENTS

Each Graduate Program specifies student milestones for satisfactory progress towards the completion of all degree requirements. All non-thesis degree requirements must be completed before the student and/or Graduate Chair can submit a proposal for the Thesis Examination Board to SGPS.

1.3 THE THESIS PREPARATION AND FORMAT

The work must comprise a coherent account of a unified research project rather than a collection of loosely connected studies. It must have an acceptable form for its discipline and display a thorough knowledge of and scholarly approach to the subject.

Before beginning to write, each student, in consultation with his/her Supervisory Committee and Graduate Program, should decide on the best format in which to present the work.

1.3.1 Formatting

Format Specifications

SGPS accepts theses in either monograph or integrated-article format:

- The monograph format organizes chapters around a central problem.
- In the integrated-article format, the chapters treat discrete but related problems. The work must have connecting materials to provide logical bridges between the different chapters, thereby achieving an integration of information. Chapters may include published articles, submitted articles, and unpublished work in a publication format. Although SGPS allows co-authorship of chapter materials, the candidate must be the principal author and have had a major role in the preparation and writing of the manuscripts. Publication or acceptance

for publication of research results before presentation of the thesis in no way supersedes the University's evaluation and judgment of the work during the thesis examination process.

- In the case of co-authored papers (chapters), the student must include a statement of authorship for each paper included in the thesis, indicating the nature and extent of contributions by others.

Monograph

- Title Page (provided by Scholarship@Western upon upload)
- Abstract and Keywords
- Co-Authorship (where applicable)
- Epigraph (optional)
- Dedication (optional)
- Acknowledgments (where applicable)
- Table of Contents
- List of Tables (where applicable)
- List of Figures (where applicable)
- List of Plates (where applicable)
- List of Appendices (where applicable)
- List of Abbreviations, Symbols, Nomenclature (where applicable)
- Preface (where applicable)
- Body of Thesis divided into various chapters and must contain:
  - Introduction
  - Literature Review (where applicable)
  - Middle sections/chapters
  - Summary, and Conclusions
  - Bibliography - Appendices (copyright releases and ethics approval should be included where applicable)
  - Curriculum Vitae

Integrated-Article

- Title Page (provided by Scholarship@Western upon upload)
- Abstract and Keywords
- Co-Authorship (where applicable)
- Epigraph (optional)
- Dedication (optional)
- Acknowledgments (where applicable)
- Table of Contents
- List of Tables (where applicable)
- List of Figures (where applicable)
- List of Plates (where applicable)
- List of Appendices (where applicable)
- List of Abbreviations, Symbols, Nomenclature (where applicable)
- Preface (where applicable)
- Body of Thesis:
  - Introductory chapter to the entire thesis with its own bibliography
  - Literature Review (where applicable)
  - Middle chapters. Each is presented in an integrated-article format without an abstract, but with its own bibliography.
  - Final chapter (general discussion and conclusions) to relate the separate studies to each other and to a relevant discipline or field of study. This section has its own bibliography.
- Appendices: This section to contain details of methodology, tabulated data, and other pertinent data not provided in detail in previous chapters. Ethics Approval (where applicable) and copyright releases from publications must be included here.

- Curriculum Vitae Note: The bibliographies for each of the individual chapters should be in a consistent format throughout the thesis regardless of the citation formats of the journals in which the article has appeared or will appear. Tables and figures should appear in the text where they would be for publication.

1.3.1.1 Length

Programs may have regulations that limit length. Students should contact their program for these regulations.

1.3.1.2 Copyrighted Material and Permissions

The candidate must ensure that the work does not contain a substantial amount of copyrighted material. Under the Copyright Act, the "fair use" provision allows the quotation of a reasonable extract of someone else's work, if properly cited. For more extensive quotation, the candidate must obtain written permission from the copyright holder(s) and include this permission in the thesis.

If the candidate wishes the work to include text that he/she has already published as a journal article or book chapter, he/she must obtain permission from the publisher and include this permission in the appendices. This is of utmost importance if the integrated-article format is used. Any chapters that have been published, accepted for publication, or submitted for publication must carry the following footnote: A version of this chapter has been published/accepted for publication/submitted for publication (Cite the reference).

1.3.1.3 Confidentiality Agreement

If the candidate feels that the nature of the information contained in the work must remain confidential (e.g., concerns pending patents etc.) for a specified period of time, a confidentiality agreement is required. See Sections 1.4.3 and 1.5.3: The Candidate Submits the Thesis for Examination.

1.3.1.4 Titling

An effective title makes the thesis accessible to other scholars. The title must provide an accurate description of the thesis content. Library catalogues and online bibliographic databases use words in the title as a way to retrieve a thesis. Thus, if possible, the title should include key words that link the thesis to literature on its topic. Use word substitutes for formulas, symbols, superscripts, subscripts, Greek letters, and so on.

1.3.1.5 Title Page

The title page contains the copyright notice and information to identify the thesis in catalogues and bibliographies. A title page is generated and attached to the front of a thesis after it is uploaded to Scholarship@Western.

1.3.1.6 Certificate of Examination

The signed Certificate of Examination will be retained in the student's file within the School of Graduate and Postdoctoral Studies.

Samples:

- Doctoral With Supervisor
  - http://grad.uwo.ca/doc/academic_services/thesis/doctoral_certificate_supervisor.doc
- Doctoral With Co-Supervisor
  - http://grad.uwo.ca/doc/academic_services/thesis/doctoral_certificate_co_supervisor.doc
- Doctoral With Joint-Supervisor
  - http://grad.uwo.ca/doc/academic_services/thesis/doctoral_certificate_joint_supervisor.doc
- Master's With Supervisor
  - http://grad.uwo.ca/doc/academic_services/thesis/masters_certificate_supervisor.doc
- Master's With Co-Supervisor
  - http://grad.uwo.ca/doc/academic_services/thesis/masters_certificate_co_supervisor.doc
- Master's With Joint-Supervisor
  - http://grad.uwo.ca/doc/academic_services/thesis/masters_certificate_joint_supervisor.doc

1.3.1.7 Abstract and Keywords

The abstract (page-numbered ii) provides a succinct summary of the work. To satisfy the requirements of the National Library and University Microfilms International, the abstract must be no longer than 350 words for a Doctoral thesis and 150 words for a Master's thesis.

The abstract must give enough information about the thesis to allow a potential reader to decide whether or not to consult the complete work. The candidate must ensure that the abstract refers to

all the elements that would make the thesis worth consulting. The abstract should include important place names and proper nouns because these can be significant key words for electronic retrieval. It should not include graphs, charts, illustrations or tables. The expected content of an abstract varies among disciplines, but all abstracts can be expected to include the following:

- a statement of the research problem or question
- an indication of the research method(s) used or theoretical orientation taken
- findings or major discoveries made
- conclusions and significance.

The abstract should have the same line spacing as the text of the body of the thesis.

Keywords: At the end of the Abstract should appear a list of keywords. Librarians use these keywords when assigning subject headings and index terms as part of the Cataloguing Record. The candidate, as the person most familiar with the research and its significance, selects the terms that help other scholars get access to his/her work. For example, the following keywords could be used for a thesis studying fuzzy relational modeling:

Keywords: fuzzy relational modeling, fuzzy simulation, fuzzy c-Means, Centre of Gravity, Centre of Area, Weighted Average of Cluster Centres, Polyline algorithm.

The following could be used to describe a thesis entitled "The Other of Grammatology: Lacan, Derrida, Kristeva," which studies the relationship between psychoanalysis, deconstruction, and feminism as they relate to linguistic representations: Keywords: Psychoanalysis, Deconstruction, Freud, Derrida, Kristeva, Lacan, Feminism, Reference.

1.3.1.8 The Co-Authorship Statement (where applicable)

In the case where a thesis includes papers co-authored by the candidate and others, the thesis must state explicitly who contributed to such work and the nature and extent of this contribution. The Supervisor(s) must attest to the accuracy of such statements about co-authorship at the Thesis Examination.

1.3.1.9 Acknowledgements (where applicable)

The acknowledgments note help received from the Supervisor(s), staff, co-authors and co-researchers, fellow students, technicians or others in the collection of materials or data, the design and construction of apparatus, the analysis of data, and the writing of the thesis.

1.3.1.10 Table of Contents

The table of contents provides a listing of the main elements in the thesis. In the rare case where the thesis requires more than one volume, each volume must have its own table of contents.

1.3.1.11 Ethics Approval (where applicable)

If the thesis has involved the use of animal or human subjects, the candidate must provide evidence of the necessary ethics approval from the appropriate committee, such as a copy of the UWO ethics approval form in an appendix.

For more information on research involving humans consult the Office of Research Ethics website (http://www.uwo.ca/research/ethics/) . For information on the use of animals consult the Animal Care and Veterinary Service website (http://www.uwo.ca/animal/website/index.htm).

1.3.1.12 Appendices (where applicable)

The purpose of an appendix is to include in the thesis supporting material that is not an essential part of the text itself. For example, in a thesis that involves a survey, letters of information to subjects, questionnaires, or other research instruments may appear in an appendix. In a thesis that analyzes a rare or inaccessible text, that text may be included in an appendix. An appendix also may include raw data on which analysis has been performed, either in print or disk format.

1.3.1.13 Curriculum Vitae

The vita should be a brief document and include only public information: name, post secondary education and degrees, awards, related work experience, and relevant publications. It is not intended to be a job resumé. The Vita is the last page(s) of the thesis.

Sample: http://grad.uwo.ca/doc/academic_services/thesis/cv_sample.pdf

1.3.1.14 Electronic Thesis and Dissertation (ETD)

Candidates must present their work in an acceptable file format type. These acceptable types are:

Portable Document Format (.pdf) Word Document (.doc) Rich Text Format (.rtf) Note: candidates are encouraged to use the Portable Document Format (.pdf) as all documents received through the Scholarship@Western Electronic Thesis and Dissertation repository are automatically converted to .pdf. Using the .pdf format allows the candidate to verify the appearance and organization of the document as it will be presented to the examiners.

1.3.1.15 Supplementary Content in the ETD

Candidates who would like to include supplementary multimedia to accompany their work may do so in the Supplementary Content section of the submission process. These files may have any format type, however candidates should be aware that the content should be in a universally accessible format – this is required in order to ensure that examiners will be able to access the material.

1.3.2 Style

1.3.2.1 Style for headings, subheadings, references, figures, tables, spelling, punctuation, and bibliographic citations

The candidate must follow a standard style manual that has the approval of his/her Graduate Program, so that the form and location of notes and the presentation of references/bibliographies is consistent throughout the thesis and conforms to a style appropriate to the discipline.

1.3.2.2 Typescript and points size

The style of font, font size, footnote/reference method, pagination, margins, and any other aspects of production are to be consistent throughout the thesis. For the text, type smaller than 12 point must not be used, but a smaller point size, not less than 9, is acceptable for footnotes, graphs, formulas, and appendices.

1.3.2.3 Line spacing

All textual material (including the abstract, acknowledgments, and other preliminary material) must have 1.5 - 2 spaces between lines. The only exceptions to this requirement are references, bibliographies, and indented long quotations, which may be single-spaced.

1.3.2.4 Margins

Due to the requirements of binding and microfiching, the candidate must observe the following specifications for margins on all copies. Leave a margin of at least 38 mm (1.5 inches) from the left-hand edge of the paper. Leave a margin of at least 25 mm (1 inch) from the top, bottom and right edges. These margins apply equally to all illustrative material: diagrams, maps, photographs, charts, tables etc. (except as noted in section 3.2.5).

1.3.2.5 Illustrative material

Illustrative material must appear in the text, not at the end of chapters. All illustrative material, from ink drawings to printed maps, charts and graphs to photographs must be readable. Annotate appropriately coloured charts, figures, graphs or maps, since the colours will reproduce in indistinguishable shades of grey on microfiche. When photographs are incorporated into the thesis, they should be high-contrast colour or black-and-white prints. Explanations or captions of figures and tables may appear beneath the figures and tables to which they refer or they may face them.

1.3.3 Pagination

Page Numbering

With the exception of the title page (provided by Scholarship@Western after upload), each page in the work must be numbered. The following system is to be used:

- The pages of preliminary material (acknowledgment, table of contents, etc.) must be numbered with small Roman numerals (i, ii, iii) placed in the centre of the page, not less than 12.5 mm (.5 inch) from the bottom edge.
- The body of the thesis, starting with the first page of the Introduction or Chapter One as page 1, must be numbered with Arabic numerals (1, 2, 3) placed in the upper righthand corner, not less than 12.5 mm (.5 inch) from each edge to avoid being cut off during the binding of paper copies.
- For page numbering of illustrative material see Section 3.2.5 Illustrative Material.

1.4 DOCTORAL CANDIDATE COMPLETION OF THE THESIS DEGREE REQUIREMENT

Normally the entire process, from the Graduate Chair's request for a Thesis Examination to the placement of the candidate's name on the convocation list, requires approximately eight weeks. For an overview of submission dates, please visit Thesis Timelines.

1.4.1 The Program Requests a Thesis Examination

When the thesis is thought to meet recognized scholarly standards for the discipline and degree and is ready for examination, the Graduate Chair arranges a Thesis Examination by setting a proposed

date, and obtaining provisional consent from the potential members of the Thesis Examination Board. (The thesis Supervisor(s), Supervisory committee or the candidate alone may also initiate this process.) He/she must then submit for approval the Examination Board to SGPS, using the Proposed Doctoral Thesis Examination Board form at least seven working weeks before the proposed date, and, where applicable, set a date and time for the Public Lecture.

1.4.2 SGPS Approves the Thesis Examination Board and Thesis Examination (and Public Lecture, If Applicable)

SGPS approves the Thesis Examination Board provided by the candidate's program. Doctoral candidates must submit the thesis six weeks before the approved date for the Thesis Examination. This ensures adequate time for:

Providing access to the thesis for the Examiners Examiners to read the thesis and prepare their reports Examiners to submit reports to SGPS Candidates are required to present a Public Lecture on their thesis research, normally within twenty-four hours before the Thesis Examination. The Graduate program sets the time and place for the lecture. SGPS announces the public lecture on its website and in the Western News. The lecture is open to all members of the community. The Examiners should normally attend the Public Lecture and Thesis Examination.

Doctoral Only - Effective May 2012, public lectures are mandatory for all programs.

The Thesis Examination and Public Lecture may be postponed or cancelled if any step in the examination process is not completed on schedule (e.g. the candidate fails to submit the Thesis for Examination on schedule, or the Examiners fail to submit evaluations on time).

Note: The thesis defense is normally a closed event unless the student and program, by mutual agreement, request that the defense be open to the university community (e.g. faculty, academic colleagues, students).

1.4.2.1 Doctoral Thesis Examination Board Roles

Doctoral Thesis Examination Board Roles

Chair:

The Chair is a non-voting member of the Thesis Examination Board. As the Vice-Provosts' (Graduate and Postdoctoral Studies) representative, the Chair presides over the thesis examination and provides leadership to ensure that the established procedures are followed. It is not appropriate for the Chair to ask the Candidate Thesis related questions during the examination period.

Chair Duties:

- Determines when a quorum exists

- Opens and closes the examination proceedings
- Sets the order of questioners and the length of their question periods
- Monitors the length and conduct of the candidate's presentation
- If the External Examiner is not present, ensures that questions raised in the External Examiner's report are put to the candidate
- If requested by the Vice-Provost (Graduate and Postdoctoral Studies), where the External Examiner has submitted a negative report but is not present, provides copies of the External Examiner's report to the Examiners to assist in their deliberations
- Intervenes if questioning becomes inappropriate
- Deals with behaviour that interferes with the proper conduct of the examination
- Moderates in camera discussion on the merits of the thesis, the candidate's oral presentation and responses to questions, the External Examiner's report, and other relevant matters
- Calls for a vote and recommendation
- Recalls the candidate and advises him/her of the recommendations that are to be made to the Vice-Provost (Graduate and Postdoctoral Studies)
- Prepares a report to the Vice-Provost (Graduate and Postdoctoral Studies) of the Examiners' assessment of the thesis and the candidate's oral performance

Chair Qualifications:

- Must have appropriate SGPS membership
- The Chair must not be a member of the candidate's program or the Supervisor's home program

Note: If, at the conclusion of the defense, the candidate's supervisor, the Chair or any member of the examining committee expresses the view that there is a prima facie case for alleging that a material portion of the thesis has been plagiarized, or that there is other evidence of academic misconduct, the Chair shall withhold his/her signature from the examination certificate and submit the matter (together with any supporting materials) to SGPS for investigation. Where this occurs, the Chair shall, without informing the candidate of the identity of the person making the relevant allegation, inform the candidate that an allegation of academic misconduct has been made. The Chair shall also inform the candidate that an investigation into the matter will be conducted.

Two Program Examiners:

- Attend the Thesis Examination and participate in the questioning of the candidate, evaluating the thesis and the candidate's responses at the oral defense
- Cast a vote in the final determination of the acceptability of the thesis and oral defense

Regulations:

- Must have appropriate SGPS membership
- Must attend the Public Lecture
- No more than one Program Examiner may be from the candidate's Thesis Supervisory Committee
- Must not have had significant involvement in the development of the thesis nor interest in the outcome

University Examiner:

- The University Examiner brings to the thesis examination insights from outside the candidate's discipline
- He/she must have knowledge in the general field of the thesis, but need not be an expert on the thesis topic
- Attend the Thesis Examination and participate in the questioning of the candidate, evaluating both the thesis and the candidate's responses at the oral defense
- Cast a vote in the final determination of the acceptability of the thesis and oral defense

Regulations:

- Must have appropriate SGPS membership
- He/she must not be a member of the candidate's Thesis Supervisory Committee, or a member of the Supervisor's home program
- Where the program unit is a Faculty, the University Examiner must not be from the candidate's or Supervisor's home Department
- Where the University Examiner is from a unit that does not have a Graduate program, or from outside the University, the Graduate Chair of the candidate's program must nominate him/her for non-core limited membership in SGPS

External Examiner:

- Have an established reputation in the special field of the thesis
- Be able to judge whether the thesis would be acceptable at another distinguished university
- Attends the Public Lecture and Thesis Examination in person (participation by videoconference or teleconference is also permitted), and participate in the questioning of the candidate, evaluating both the thesis and the candidate's responses at the oral defense
- Casts a vote in the final determination of the acceptability of the thesis and oral defense

Regulations:

- Is normally a faculty member from another university
- Must not be associated or affiliated with UWO
- Must be at arm's-length (see below)

Arm's-Length of Examiners

Examiners must be seen to be able to examine the student and the thesis at arm's-length, free of substantial conflict of interest from any source. The test of whether or not a conflict of interest might exist is whether a reasonable outside person could consider a situation to exist that could give rise to an apprehension of bias. Co-authors or collaborators of any component of the thesis may not serve as Examiners.

Relationships that might appear to have a conflict of interest include:

- The involvement of an Examiner with the candidate or Supervisor in a personal capacity, such as:
  - A spouse or partner
  - A close family member
  - A business partner
  - Having previous, current, or future negotiations relating to employment
- This list, while not exhaustive, illustrates the nature of potential conflicts to be avoided. The candidate's program must take reasonable steps to avoid recommending an Examiner whose relationship with the candidate or Supervisor could be seen as jeopardizing an impartial judgment on the thesis. Best practices include reviewing the potential examiner's CV; having the grad committee members review the list of names nominated as examiners; conducting a literature search on potential examiner's publications. It is recommended that supervisors and programs avoid multiple use of the same examiners.

- A faculty member asked to examine a thesis should declare possible sources of conflict.

1.4.3 The Candidate Submits the Thesis for Exmination

No later than six weeks before the date of the Thesis Examination, the Doctoral candidate submits a copy of their work for preliminary examination. This is done through digital submission via the Scholarship@Western Electronic Thesis and Dissertation Repository.

A completed Doctoral Thesis Supervisor Approval form must be submitted directly to SGPS. In those cases where the student chooses to submit a Thesis for Examination without the approval of the Supervisor(s), the Supervisor(s) must state on the Doctoral Thesis Supervisor Approval form why his/her approval is withheld. The Graduate Chair signs the form and provides the candidate with a copy of the Supervisor's stated reasons for withholding approval.

Once the thesis has been officially submitted for examination, it cannot be withdrawn except with the permission of the Vice-Provost (Graduate and Postdoctoral Studies).

1.4.3.1 Confidentiality Agreement

If the candidate feels a confidentiality agreement is required, the candidate must ensure that each Examiner's signed agreement is delivered to the School of Graduate and Postdoctoral Studies along with the Doctoral Thesis Supervisor Approval form. The School of Graduate and Postdoctoral Studies will ensure the Chair of the Thesis Examination has signed an agreement prior to the Thesis Examination.

1.4.3.2 Delay of Publication

Note: please see section 1.4.6 for important information regarding the electronic publication of theses.

If you need to delay publication of your thesis or dissertation, you can indicate an automatic "delay of publication", for up to two years, on your work. This option will block your work from public access after your successful examination and final submission. This process is available as part of the Scholarship@Western Electronic Thesis and Dissertation submission process. When the "delay of publication" expires, the author can be granted a one-year extension through a written request to the Thesis Coordinator.

Exceptionally, a candidate may request a six-year delay of publication by contacting an Associate Vice-Provost within the School of Graduate and Postdoctoral Studies. This request requires the approval of the Policy, Regulations and Graduate Program Membership Committee of the Graduate Education Council.

1.4.4 The Examination of the Thesis and the Candidate

The Tasks of the Examiners are to:

Determine if the thesis and the candidate meet recognized scholarly standards for the degree Appraise the thesis for content - its underlying assumptions, methodology, findings, and scholarly significance of the findings Appraise the thesis for form - its organization, presentation of graphs, tables, and illustrative materials, and its use of accepted conventions for addressing the scholarly literature Evaluate the candidate's skill and knowledge in responding to questions and defending the thesis Ensure authenticity of authorship SGPS distributes to the Examiners an electronic package via e-mail consisting of:

A formal electronic invitation to examine the thesis and the candidate The date, time and location of the examination Instructions on how to access the Scholarship@Western ETD repository The Thesis, in .PDF format available through the Scholarship@Western ETD repository The option to request the thesis in a paper format through Graphic Services Pertinent excerpts from the Thesis Regulation Guide The secure Thesis Examiner Report available through the Scholarship@Western ETD repository For the External Examiner, please visit External Examiners for appropriate forms and information. The Examiners do their work in a two-stage process.

1.4.4.1 Stage One : The Preliminary (or Pre-Examination) Evaluation of the Thesis

Each Examiner must independently and without consultation, decide whether the thesis meets the scholarly standards for the discipline and degree in form and content.

There are 2 possible outcomes that the examiners may consider:

1. Acceptable to go to defense with revisions/modifications

    - **Acceptable with Revisions/Modifications**: A work that requires some revisions/modifications may be judged acceptable. Revisions/modifications include limited typographical or grammatical errors; errors in calculation, labels for tables, nomenclature, and bibliographic form; and the need for clarification of content.

2. Unacceptable to go forward to defense

    - **Unacceptable**: A thesis judged unacceptable may contain for example, faulty conceptualization, inappropriate or faulty use of research methodology, misinterpretation or misuse of data, neglect of relevant material, illogical argument, unfounded conclusions, seriously flawed writing and presentation, and failure to engage the scholarly context.

The completed examiner reports are confidential to the Vice-Provost (Graduate and Postdoctoral Studies). The External Examiner completes the External Examiner's Report. SGPS must receive

the completed forms from all the Examiners at least five working days before the date scheduled for the candidate's Thesis Examination. All examiner evaluations are shared with the Supervisor after the examination.

If the Thesis Content and Thesis Form is Judged Acceptable

A majority of the Examiners must judge that both the thesis content and thesis form are acceptable to allow Stage Two: The Thesis Examination to proceed. An Examiner's preliminary judgment of acceptability is provisional. It does not preclude the Examiner changing his/her judgment to finding the thesis unacceptable at the Thesis Examination.

If the Thesis Content and Thesis Form is Judged Unacceptable

If there is not a majority of Examiners who judge the thesis to be acceptable in both content and form, SGPS cancels the Thesis Examination, and the Vice-Provost (Graduate and Postdoctoral Studies) appoints a Re-Submission Hearing Committee which reviews the case and decides whether or not to allow the candidate to prepare a revised version of the thesis for examination and, if so, the time limit for doing so.

- Where the Re-Submission Hearing Committee* agrees that the candidate should be given the opportunity to revise the thesis to bring it to the acceptable scholarly standard for examination, the committee first establishes a new Thesis Examination date, no earlier than twelve weeks after the date for the originally scheduled examination. Normally the same Examiners assess the re-submitted thesis. The Chair of the committee provides the Graduate Chair, Supervisor(s), and student written notification of the decision and the changes suggested by the committee.

  *The Re-Submission Hearing Committee is chaired by an Associate Vice-Provost (Graduate and Postdoctoral Studies) and includes the two Program Examiners, University examiner (the External Examiner's presence is waived), the Supervisor, the Graduate Chair of the program concerned. Note: The candidate does not attend the committee meeting.

- The Examination must then proceed as listed in Section 4.1 The Program Requests a Thesis Examination above for the re-submitted thesis.

- Whether or not the re-submitted thesis is found acceptable, the candidate proceeds to Stage Two: The Thesis Examination.

1.4.4.2 Stage Two: The Thesis Examination

The Chair presides over the Thesis Examination:

- To open proceedings, the Chair introduces all present.

  - The candidate, the Supervisor(s), the Program Examiners, and the University Examiner must attend the Thesis Examination
  - SGPS prefers that the External Examiner attend, however the Vice-Provost (Graduate and Postdoctoral Studies) may waive the presence of the External Examiner. If unable to attend, the External Examiner must submit questions to be put to the candidate by the other Examiners. Alternatively, the External Examiner may allow the other Examiners access to his/her report immediately before the Thesis Examination so that they can question the candidate on the issues it raises.
  - Any member of SGPS may attend as a visitor by having a written request to attend approved by the Vice-Provost (Graduate and Postdoctoral Studies)
  - The Chair will refuse attendance to all others

- Examiners are asked to refrain from using electronic devices (cell phones) during the examination (unless in emergencies)

- The Chair then asks the candidate (and visitors) to leave the room so that the Examiners can decide on the following points:

  - The order in which Examiners are to question the candidate;
  - The number of rounds of questioning desired (usually two);
  - The time limit for each of the Examiners' questioning periods (typically 15-20 minutes in the first round and 5-10 minutes in the second round);
  - Who is to ask the questions submitted by the External Examiner if he/she is not present;

- The Chair gives to each Examiner a Doctoral Thesis Examination Evaluation form (to be completed when the questioning of the candidate is over and the candidate has left the room). The Chair advises the Examiners that their evaluations on acceptability of the thesis content and thesis form should be made independent of the assessment made in Stage One: The Preliminary Evaluation of the Thesis.

- The Chair invites the candidate (and visitors) back into the room.

The Examination Begins

- The Chair explains to the candidate the sequence of events (e.g. two rounds of questioning, order of questioning).

- Where the candidate's program does not provide for a public lecture, the candidate may briefly discuss the thesis (10-15 minutes is appropriate).

- The Examiners question the candidate in the agreed-upon order, with the Chair holding them to the agreed-upon time limit. The Supervisor(s) may not question the candidate.
- When the questioning has finished, the Chair asks the candidate and visitors, but not the Supervisor(s), to leave the room.

The Thesis Examination Board Deliberates and Renders a Decision

- The Chair invites the Supervisor(s) to comment on the candidate, the thesis, and aspects of the oral defense.
  - In rare cases where the thesis has been submitted without the Supervisor(s)'s approval, the Chair informs the Examiners of the Supervisor(s)'s written reasons for withholding approval, before inviting the Supervisor(s) to speak.
- At the Chair's invitation, the Examiners alone discuss the thesis and the oral defense.
- The Chair instructs the Examiners once again about the difference between acceptable, acceptable with revisions/modifications and unacceptable and answers any questions about the difference between the recommendation. (See Section 1.4.4.1 Stage One: The Preliminary Evaluation of the Thesis)
- The Examiners vote on the acceptability of the thesis and the oral defense by completing their Doctoral Thesis Examination Evaluation form. In cases where the External Examiner is not physically present, the Chair speaks to her/him privately and fills out the Evaluation form as directed.
- These forms are confidential, only to be seen and recorded by the Chair. For the oral defense, the Examiners must determine if the candidate's responses to questions and general level of scholarly knowledge meets the standard for the Doctoral degree and is consistent with the contents of the thesis. The Examiners must decide whether the thesis form and thesis content and oral defense were acceptable or unacceptable.
- There are 3 possible outcomes to the oral defense that the examiners may consider:
  - Acceptable - no changes
  - Acceptable with revisions/modifications
    - **Examples of Acceptable with Revisions/Modifications**: May include limited typographical or grammatical errors; errors in calculation, labels for tables, nomenclature, and bibliographic form; the need for clarification of content in order to

meet requisite scholarly standards. Examples may include some additions, deletions or editing of text; further analysis or discussion of some piece of data. Normally, candidates have up to 6 weeks to submit the final thesis after examination.

  - Unacceptable
    * Unacceptable: A thesis judged unacceptable may contain for example, faulty conceptualization, inappropriate or faulty use of research methodology, misinterpretation or misuse of data, neglect of relevant material, illogical argument, unfounded conclusions, seriously flawed writing and presentation, and failure to engage the scholarly context.

- The Chair collects the completed forms and tallies the results.
- The Chair announces the results of the vote on the acceptability of the content and form of the thesis and of the oral defense and asks if further discussion is needed. In rare instances the Chair may allow Examiners to change their votes.
- If a majority of the Examiners find that each of the thesis content, thesis form, and the oral defense are acceptable, the candidate passes the Thesis Examination. The Examiners' approval may be conditional on the candidate successfully completing revisions to the thesis content or thesis form.
- If the examiners decisions are equally split (2/2) between acceptable and unacceptable on any one of the thesis content, thesis form and/or the oral defense, then the vote is weighted in favour of the external examiner's decision.
- The Chair pronounces the Thesis Examination Board's decision.

When the Thesis Examination is Successful

- On the "Doctoral Thesis Examination - Chair Report," the Chair:
  - Reports the Thesis Examination Board's decision for the Thesis Examination on page 1
  - With the assistance of the Examiners, lists the revisions/modifications, if any, required by a majority of the Examiners on page 2.
  - Dates and signs both pages of the Chair's Report.
- In the case that no revisions/modifications are required, the Chair, the Examiners, and the Supervisor(s) sign the Certificate of Examination. In the case that revisions/modifications are

required, the Chair, with the help of the committee, determines who will withhold her/his signature(s) until the required revisions/modifications have been made.

- The Chair communicates the positive decision to the candidate. (See Communicating the Decision to the Candidate.)

When the Thesis Examination is Unsuccessful

The Chair completes the "Doctoral Thesis Examination - Chair Report." In consultation with the Examiners, the Chair states (on page 2 of the report) why the thesis and/or the oral defense was unacceptable.

If the Thesis Examination failed because the thesis content and/or thesis form was unacceptable, the Vice-Provost (Graduate and Postdoctoral Studies) refers the case to a Re-Submission Hearing Committee.

- Where the Re-Submission Hearing Committee agrees that the candidate should be given the opportunity to revise the thesis to bring it to the acceptable scholarly standard for examination, the committee first establishes a new Thesis Examination date, no earlier than twelve weeks after the date for the originally scheduled examination. Normally the same Examiners assess the re-submitted thesis. The Chair of the committee provides the Graduate Chair, Supervisor(s), and student written notification of the decision and the changes suggested by the committee.

- The Examination must then proceed as listed in Section 4.1 The Program Requests a Thesis Examination for the re-submitted thesis.

- Whether or not the re-submitted thesis is found acceptable, the candidate proceeds to Stage Two: The Thesis Examination.

If the Thesis Examination failed solely because of an unacceptable oral defense, the Vice-Provost (Graduate and Postdoctoral Studies) refers the case to a Re-Examination Hearing Committee.

- Where the Re-Examination Hearing Committee* decides that a second Thesis Examination is appropriate, it should be held, preferably with the same Thesis Examination Board, within a time period determined by the committee. The Chair of the committee provides the Graduate Chair, Supervisor(s), and student written notification of the decision.

  * The Re-Examination Hearing Committee is chaired by an Associate Vice-Provost (Graduate and Postdoctoral Studies) and includes the two Program Examiners, University Examiner (the External Examiner's presence is waived), the Supervisor, and the Graduate Chair of the program concerned. Note: The candidate does not attend the committee meeting.

If the candidate has already been through a Re-Submission Hearing Committee or a Re-Examination Hearing Committee (following Stage Two: The Thesis Examination), then the Thesis Examination Board's decision is final. The candidate has no further opportunity for resubmission and/or re-examination. However, if a candidate has been through a Re-Submission Hearing Committee following Stage One: The Preliminary (or Pre-Examination) Evaluation of the Thesis, then the Vice-Provost (Graduate and Postdoctoral Studies) refers the case to a second and final Re-Submission Hearing Committee.

Communicating the Decision to the Candidate

When the Chair and the Examiners have completed the documentation, the Chair invites only the candidate back into the room and informs him/her of the result.

- Following a positive decision that is subject to revisions/modifications, the Supervisor(s) must meet with the candidate, to ensure that he/she understands the revisions/modifications required by the Thesis Examination Board.

Following the Examination

The Chair of the examination returns all forms to the School of Graduate and Postdoctoral Studies.

1.4.5 Final Submission of the Thesis

When the candidate has completed any changes recommended by the Thesis Examination Board the candidate must submit the final copy of their work via digital submission through the Scholarship@Western Electronic Thesis and Dissertation Repository.

The candidate accesses their original submission within the repository and submits a revised copy of their work.

Candidates are required to submit the following to the School of Graduate and Postdoctoral Studies before publication can occur:

- A National Library Non-Exclusive License (scanned copies is acceptable)
- The signed Certificate of Examination (scanned copies is acceptable)

Once the thesis is published, the candidate has officially completed the thesis requirement for their degree. Subject to approval by the University Senate, the candidate's name is placed on the convocation list. Should a thesis writing candidate require proof of completion of all degree requirements he/she should contact the Thesis Coordinator.

1.4.6 Publishing of the Thesis

A thesis is a research document. It is a record of the research that the student conducted while completing a graduate degree. Given the research and education mandate of Canada's publicly funded universities, it is expected that the results of this research will be made publicly available.

For those theses that are submitted through the Scholarship@Western ETD Repository, the University requires that successful graduate theses be made available through the Western Library's Scholarship@Western program. The thesis will be published electronically at the conclusion of the degree process, and will be available globally via the World Wide Web.

At the same time, the University recognizes that the student is the author of the thesis, and retains copyright and control interests in the material.

Note: Students should be conscious of the implications of electronic publication in the digital context: material is accessible to any interested party, academic and non-academic. The thesis should also be understood to be permanently available – once published electronically, it can be withdrawn from Scholarship@Western, but digital copies will inevitably persist. Students engaging in thesis preparation and research should be mindful of electronic publication and availability as an endpoint of their research. Supervisors, equally, have a responsibility to be acquainted with the implications of electronic publication, and advise their students accordingly.

In certain cases, a "delay of publication" may be appropriate. See section 1.4.3.2 above.

The candidate is asked to permit the release of the thesis to be used for research by signing the "National Library Non-Exclusive License" to allow the non-exclusive right to reproduce or loan copies of the thesis in micro-form, paper, or electronic formats. The term "non-exclusive license" means that the author retains the copyright of the thesis and can seek other forms of publication.

Upon final approved submission, the work is published to the Scholarship@Western ETD repository, pending any requests for a delay of publication. This repository is publicly accessible, permitting free access to the work. The repository transmits regular reports via e-mail to the author on how often the work is accessed.

1.4.7 Archiving of the Thesis

Western preserves all doctoral theses in microform images within microfiche flat sheets formatting. These archival copies are stored within Western Libraries. Other format versions of the thesis (e.g. paper, digital) may also be retained.

1.5 MASTER'S CANDIDATE COMPLETION OF THE THESIS DEGREE REQUIREMENT

1.5.1 The Program Requests a Thesis Examination

When the thesis is thought to meet recognized scholarly standards for the discipline and degree and is ready for examination, the Graduate Chair arranges a Thesis Examination by setting a proposed date, and obtaining provisional consent from the potential members of the Thesis Examination Board. (The thesis Supervisor(s), Supervisory committee or the candidate alone may also initiate this process.) He / she must then submit for approval the Examination Board to SGPS [PDF] at least four working weeks before the proposed date.

1.5.2 SGPS Approves the Thesis Examination Board and Thesis Examination (and Public Lecture, If Applicable)

SGPS approves the Thesis Examination Board and the date of the examination. The date and time of the examination will be confirmed within the formal invitation from SGPS. Master's candidates must submit the thesis three working weeks before the approved date for the Thesis Examination. This ensures adequate time for:

- Providing access to the thesis for the Examiners
- Examiners to read the thesis and prepare their reports
- Examiners to submit reports to SGPS

The Thesis Examination may be postponed or cancelled if any step in the examination process is not completed on schedule (e.g. the candidate fails to submit the Thesis for Examination on schedule, or the Examiners fail to submit evaluations on time).

Note: The thesis defense is normally a closed event unless the student and program, by mutual agreement, request that the defense be open to the university community (e.g. faculty, academic colleagues, students).

1.5.2.1 The Thesis Examination Board

Master's Thesis Examination Board Chair

Chair:

The Chair is a non-voting member of the Thesis Examination Board. As the Vice-Provosts' (Graduate and Postdoctoral Studies) representative, the Chair presides over the thesis examination

and provides leadership to ensure that the established procedures are followed. It is not appropriate for the Chair to ask the Candidate Thesis related questions during the examination period.

Chair Duties:

- Determines when a quorum exists
- Opens and closes the examination proceedings
- Sets the order of questioners and the length of their question periods
- Monitors the length and conduct of the candidate's presentation
- If the External Examiner is not present, ensures that questions raised in the External Examiner's report are put to the candidate
- If requested by the Vice-Provost (Graduate and Postdoctoral Studies), where the External Examiner has submitted a negative report but is not present, provides copies of the External Examiner's report to the Examiners to assist in their deliberations
- Intervenes if questioning becomes inappropriate
- Deals with behaviour that interferes with the proper conduct of the examination
- Moderates in camera discussion on the merits of the thesis, the candidate's oral presentation and responses to questions, the External Examiner's report, and other relevant matters
- Calls for a vote and recommendation
- Recalls the candidate and advises him/her of the recommendations that are to be made to the Vice-Provost (Graduate and Postdoctoral Studies)
- Prepares a report to the Vice-Provost (Graduate and Postdoctoral Studies) of the Examiners' assessment of the thesis and the candidate's oral performance

Chair Qualifications:

- Must have appropriate SGPS membership

Note: If, at the conclusion of the defense, the candidate's supervisor, the Chair or any member of the examining committee expresses the view that there is a prima facie case for alleging that a material portion of the thesis has been plagiarized, or that there is other evidence of academic misconduct, the Chair shall withhold his/her signature from the examination certificate and submit the matter (together with any supporting materials) to SGPS for investigation. Where this occurs,

the Chair shall, without informing the candidate of the identity of the person making the relevant allegation, inform the candidate that an allegation of academic misconduct has been made. The Chair shall also inform the candidate that an investigation into the matter will be conducted.

Two Program Examiners:

Role:

- Attend the Thesis Examination and participate in the questioning of the candidate, evaluating the thesis and the candidate's responses at the oral defense
- Cast a vote in the final determination of the acceptability of the thesis and oral defense

Regulations:

- Must have appropriate SGPS membership
- Must attend the Public Lecture
- No more than one Program Examiner may be from the candidate's Thesis Supervisory Committee
- Must not have had significant involvement in the development of the thesis nor interest in the outcome

University Examiner:

Role:

- The University Examiner brings to the thesis examination insights from outside the candidate's discipline
- He/she must have knowledge in the general field of the thesis, but need not be an expert on the thesis topic
- Attend the Thesis Examination and participate in the questioning of the candidate, evaluating both the thesis and the candidate's responses at the oral defense
- Cast a vote in the final determination of the acceptability of the thesis and oral defense

Regulations:

- Must have appropriate SGPS membership
- He/she must not be a member of the candidate's Thesis Supervisory Committee, or a member of the Supervisor's home program

- Where the program unit is a Faculty, the University Examiner must not be from the candidate's or Supervisor's home Department
- Where the University Examiner is from a unit that does not have a Graduate program, or from outside the University, the Graduate Chair of the candidate's program must nominate him/her for non-core limited membership in SGPS

Arm's-Length of Examiners

Examiners must be seen to be able to examine the student and the thesis at arm's-length, free of substantial conflict of interest from any source. The test of whether or not a conflict of interest might exist is whether a reasonable outside person could consider a situation to exist that could give rise to an apprehension of bias.

Co-authors or collaborators of any component of the thesis may not serve as Examiners.

Relationships that might appear to have a conflict of interest include:

- The involvement of an Examiner with the candidate or Supervisor in a personal capacity, such as:
  - A spouse or partner
  - A close family member
  - A business partner
  - Having previous, current, or future negotiations relating to employment

This list, while not exhaustive, illustrates the nature of potential conflicts to be avoided. The candidate's program must take reasonable steps to avoid recommending an Examiner whose relationship with the candidate or Supervisor could be seen as jeopardizing an impartial judgment on the thesis. Best practices include reviewing the potential examiner's CV; having the grad committee members review the list of names nominated as examiners; conducting a literature search on potential examiner's publications. It is recommended that supervisors and programs avoid multiple use of the same examiners. A faculty member asked to examine a thesis should declare possible sources of conflict.

1.5.3 The Candidate Submits the Thesis for Exmination

No later than three weeks before the date of the Thesis Examination, the Master's candidate submits a final draft of the thesis for preliminary examination. Thi is done through digital submission via the Scholarship@Western Electronic Thesis and Dissertation Repository:

- In addition to the thesis, the candidate must ensure the following completed documents are forwarded to the School of Graduate and Postdoctoral Studies:
  - Master's Thesis Examination Board
  - Master's Thesis Supervisor Approval

In those rare cases when the student chooses to submit a Thesis for Examination without the approval of the Supervisor(s), the Supervisor(s) must state on the Master's Thesis Supervisor Approval form why his/her approval is withheld. The Graduate Chair signs the form and provides the candidate with a copy of the Supervisor(s)'s stated reasons for withholding approval.

Once the thesis has been officially submitted for examination, it cannot be withdrawn except with the permission of the Vice-Provost (Graduate and Postdoctoral Studies).

1.5.3.1 Confidentiality Agreement

If the candidate feels a confidentiality agreement (http://grad.uwo.ca/current_students/thesis/forms.html) is required, the candidate must ensure that each Examiner's signed agreement is delivered to the School of Graduate and Postdoctoral Studies along with the Master's Thesis Supervisor Approval form. The Graduate Program will ensure the Chair of the Thesis Examination has signed an agreement prior to the Thesis Examination.

1.5.3.2 Delay of Publication

Note: please see section 1.5.6 for important information regarding the electronic publication of theses.

If you need to delay publication of your thesis or dissertation, you can indicate an automatic "delay of publication", for up to two years, on your work. This option will block your work from public access after your successful examination and final submission. This process is available as part of the Scholarship@Western Electronic Thesis and Dissertation submission process. When the "delay of publication" expires, the author can be granted a one-year extension through a written request to the Thesis Coordinator.

Exceptionally, a candidate may request a six-year delay of publication by contacting an Associate Vice-Provost within the School of Graduate and Postdoctoral Studies. This request requires the approval of the Policy, Regulations and Graduate Program Membership Committee of the Graduate Education Council.

1.5.4 The Examination of the Thesis and the Candidate

The Tasks of the Examiners are to:

- Determine if the thesis and the candidate meet recognized scholarly standards for the degree
- Appraise the thesis for content - its underlying assumptions, methodology, findings, and scholarly significance of the findings
- Appraise the thesis for form - its organization, presentation of graphs, tables, and illustrative materials, and its use of accepted conventions for addressing the scholarly literature
- Evaluate the candidate's skill and knowledge in responding to questions and defending the thesis
- Ensure authenticity of authorship

SGPS distributes to the Examiners an electronic package via e-mail consisting of:

- A formal invitation to examine the thesis and the candidate
- The date, time and location of the examination
- Instructions on how to access the Scholarship@Western ETD repository
- The thesis, in .PDF format available through the Scholarship@Western ETD repository
- The option to request the thesis in a paper format through Graphic Services
- Pertinent excerpts from the Thesis Regulation Guide
- The secure Thesis Examiner Report available through the Scholarship@Western ETD repository
- For the External Examiner, please visit External Examiners for appropriate forms and information.

The Examiners do their work in a two-stage process.

1.5.4.1 Stage One: The Preliminary (or Pre-Examination) Evaluation of the Thesis

Each Examiner must independently and without consultation, decide whether the thesis meets the scholarly standards for the discipline and degree in form and content.

There are 2 possible outcomes that the examiners may consider:

1. Acceptable to go to defense with revisions/modifications

- **Acceptable with Revisions/Modifications**: A work that requires some revisions/modifications may be judged acceptable. Revisions/modifications include limited typographical or grammatical errors; errors in calculation, labels for tables, nomenclature, and bibliographic form; and the need for clarification of content.

2. Unacceptable to go forward to defense

    - **Unacceptable**: A thesis judged unacceptable may contain for example, faulty conceptualization, inappropriate or faulty use of research methodology, misinterpretation or misuse of data, neglect of relevant material, illogical argument, unfounded conclusions, seriously flawed writing and presentation, and failure to engage the scholarly context.

The completed examiner reports are confidential to the Vice-Provost (Graduate and Postdoctoral Studies). SGPS must receive the completed forms from all the Examiners at least five working days before the date scheduled for the candidate's Thesis Examination. All examiner evaluations are shared with the Supervisor after the examination.

If the Thesis Content and Thesis Form is Judged Acceptable

A majority of the Examiners must judge that both the thesis content and thesis form are acceptable to allow Stage Two: The Thesis Examination to proceed. An examiner's preliminary judgment of acceptability is provisional. It does not preclude the examiner changing his/her judgment to finding the thesis unacceptable at the Thesis Examination.

- For those reports collected via the ETD repository, SGPS will forward the preliminary decision to the candidate, examiners, supervisor(s), graduate chair and graduate assistant.

If the Thesis Content and Thesis Form is Judged Unacceptable

If there is not a majority of Examiners who judge the thesis to be acceptable in both content and form, the Program cancels the Thesis Examination, and the Graduate Chair of the Program concerned [Modified Sept. 29, 2004] appoints a Re-Submission Hearing Committee* which reviews the case and decides whether or not to allow the candidate to prepare a revised version of the thesis for examination and, if so, the time limit for doing so.

* The Re-Submission Hearing Committee is chaired by the Graduate Chair of the program concerned and includes the three Examiners, the Supervisor, and others specified in the procedures of the program concerned. Note: The candidate does not attend the committee meeting.

- Where the Re-Submission Hearing Committee agrees that the candidate should be given the opportunity to revise the thesis to bring it to the acceptable scholarly standard for examination, the committee first establishes a new Thesis Examination date, no earlier than six weeks after the date for the originally scheduled examination. Normally the same Examiners assess

the re-submitted thesis. The Chair of the committee provides the Supervisor(s) and student written notification of the decision and the changes suggested by the committee.

- The Examination must then proceed as listed in Section 1.5.1 - The Program Requests a Thesis Examination for the re-submitted thesis.
- Whether or not the re-submitted thesis is found acceptable, the candidate proceeds to Stage Two: The Thesis Examination.

1.5.4.2 Stage Two: The Thesis Examination

Who attends the Master's Thesis Examination

- The candidate, the Supervisor(s), the Program Examiners, and the University Examiner must attend the Thesis Examination
- Any member of SGPS may attend as a visitor by having a written request approved by the Vice-Provost (Graduate and Postdoctoral Studies) seven days before the examination date
- The chair will refuse attendance to all others

The Chair presides over the Thesis Examination

- To open proceedings, the Chair introduces all present.
- Examiners are asked to refrain from using electronic devices (cell phones) during the examination (unless in emergencies)
- The Chair then asks the candidate and visitors to leave the room so that the Examiners can decide on the following points:
  - the order in which Examiners are to question the candidate
  - the number of rounds of questioning desired (usually two)
  - the time limit for each of the Examiners' questioning periods (typically 15-20 minutes in the first round and 5-10 minutes in the second round);
- The Chair gives to each examiner a "Master's Thesis Examination Evaluation Form" (to be completed when the questioning of the candidate is over and the candidate has left the room). The Chair advises the Examiners that their evaluations on acceptability of the thesis content and thesis form should be made independent of the assessment made in Stage One: The Preliminary Evaluation of the Thesis.

- The Chair invites the candidate and visitors back into the room.

The Examination Begins

- The Chair explains to the candidate the sequence of events (e.g. two rounds of questioning, order of questioning).
- The Examiners question the candidate in the agreed-upon order, with the Chair holding them to the agreed-upon time limit. The Supervisor(s) may not question the candidate.
- When the questioning has finished, the Chair asks the candidate and visitors, but not the Supervisor(s), to leave the room.

The Thesis Examination Board Deliberates and Renders a Decision

- The Chair invites the Supervisor(s) to comment on the candidate, the thesis, and aspects of the oral defense.
  - In rare cases where the thesis has been submitted without the Supervisor(s)'s approval, the Chair informs the Examiners of the Supervisor(s)'s written reasons for withholding approval, before inviting the Supervisor(s) to speak.
- At the Chair's invitation, the Examiners alone discuss the thesis and the oral defense.
- The Chair instructs the Examiners once again about the difference between acceptable, acceptable with revisions/modifications and unacceptable and answers any questions about the difference between the recommendation. (See Section 1.5.4.1 Stage One: The Preliminary Evaluation of the Thesis)
- The Examiners vote on the acceptability of the thesis and the oral defense by completing their "Master's Thesis Examination Evaluation Form." These forms are confidential, only to be seen and recorded by the Chair.

The Examiners must decide whether the thesis form and thesis content and oral defense were acceptable or unacceptable. For each of thesis form and thesis content, they must find the thesis:

- acceptable - no changes
- acceptable with revisions/modifications
  - Examples of Acceptable with Revisions/Modifications: May include limited typographical or grammatical errors; errors in calculation, labels for tables, nomenclature, and bibliographic form; the need for clarification of content in order to meet requisite

scholarly standards. Examples may include some additions, deletions or editing of text; further analysis or discussion of some piece of data. Normally, candidates have up to 6 weeks to submit the final thesis after examination.

- unacceptable.
  - Unacceptable: A thesis judged unacceptable may contain for example, faulty conceptualization, inappropriate or faulty use of research methodology, misinterpretation or misuse of data, neglect of relevant material, illogical argument, unfounded conclusions, seriously flawed writing and presentation, and failure to engage the scholarly context.

For the oral defense, the Examiners must determine if the candidate's responses to questions and general level of scholarly knowledge meets the standard for the Master's degree and is consistent with the contents of the thesis.

- The Chair collects the completed forms and tallies the results.
- The Chair announces the results of the vote on the acceptability of the content and form of the thesis and of the oral defense and asks if further discussion is needed. In rare instances the Chair may allow Examiners to change their votes.
- The Chair pronounces the Thesis Examination Board's decision.
- If a majority of the Examiners find that each of the thesis content, thesis form, and the oral defense are acceptable, the candidate passes the Thesis Examination. The Examiners' approval may be conditional on the candidate successfully completing revisions/modifications to the thesis content or thesis form.

When the Thesis Examination is Successful

- On the "Master's Thesis Examination - Chair Report" the Chair:
  - Reports the Thesis Examination Board's decision for the Thesis Examination on page 1.
  - With the assistance of the Examiners, lists the revisions/modifications, if any, required by a majority of the Examiners on page 2.
  - Dates and signs both pages of the Chair's Report. In the case that no revisions/modifications are required, the Chair, the Examiners, and the Supervisor(s) sign the Certificate of Examination.

- In the case that revisions/modifications are required, the Chair, with the help of the committee, determines who will withold her/his signature(s) until the required revisions/modifications have been made.
- The Chair communicates the positive decision to the candidate. (See Communicating the Decision to the Candidate.)

When the Thesis Examination is Unsuccessful

- The Chair completes the "Master's Thesis Examination - Chair Report." In consultation with the Examiners, the Chair states (on page 2 of the report) why the thesis and/or the oral defense was unacceptable
- If the Thesis Examination failed because the thesis content and/or thesis form was unacceptable, the Vice-Provost (Graduate and Postdoctoral Studies) refers the case to a Re-Submission Hearing Committee.
- Where the Re-Submission Hearing Committee agrees that the candidate should be given the opportunity to revise the thesis to bring it to the acceptable scholarly standard for examination, the committee first establishes a new Thesis Examination date, no earlier than twelve weeks after the date for the originally scheduled examination. Normally the same Examiners assess the re-submitted thesis. The Chair of the committee provides the Graduate Chair, Supervisor(s), and student written notification of the decision and the changes suggested by the committee
- The Examination must then proceed as listed in Section 5.1 The Program Requests a Thesis Examination for the re-submitted thesis
- Whether or not the re-submitted thesis is found acceptable, the candidate proceeds to Stage Two: The Thesis Examination

If the Thesis Examination failed solely because of an unacceptable oral defense, the Vice-Provost (Graduate and Postdoctoral Studies) refers the case to a Re-Examination Hearing Committee:

- Where the Re-Examination Hearing Committee* decides that a second Thesis Examination is appropriate, it should be held, preferably with the same Thesis Examination Board, within a time period determined by the committee. The Chair of the committee provides the Graduate Chair, Supervisor(s), and student written notification of the decision
- *Re-Examination Hearing Committee is chaired by the Graduate Chair of the program concerned and includes the three Examiners, the supervisor, and others specified in the procedures of the program concerned. Note: The candidate does not attend the committee meeting

If the candidate has already been through a Re-Submission Hearing Committee or a Re-Examination Hearing Committee (following Stage Two: The Thesis Examination), then the Thesis Examination Board's decision is final. The candidate has no further opportunity for resubmission and/or re-examination. However, if a candidate has been through a Re-Submission Hearing Committee following Stage One: The Preliminary (or Pre-Examination) Evaluation of the Thesis, then the Vice-Provost (Graduate and Postdoctoral Studies) refers the case to a second and final Re-Submission Hearing Committee.

Communicating the Decision to the Candidate

When the Chair and the Examiners have completed the documentation, the Chair invites only the candidate back into the room and informs him/her of the result.

- Following a positive decision that is subject to revisions/modifications, the Supervisor(s) must meet with the candidate, to ensure that he/she understands the revisions/modifications required by the Thesis Examination Board

Following the Examination

The Chair of the examination returns all forms to the graduate program office.

1.5.5 Final Submission of the Thesis

When the candidate has completed any changes recommended by the Thesis Examination Board the candidate must submit the final copy of their work via digital submission through the Scholarship@Western Electronic Thesis and Dissertation Repository:

The candidate accesses their original submission within the repository and submits a revised copy of their work.

Candidates are required to submit the following to the School of Graduate and Postdoctoral Studies before publication can occur:

- The signed Certificate of Examination (scanned copy is acceptable)

Once the thesis is published, the candidate has officially completed the thesis requirement for their degree. Subject to approval by the University Senate, the candidate's name is placed on the convocation list. Should a thesis writing candidate require proof of completion of all degree requirements he/she should contact the Thesis Coordinator.

1.5.6 Publishing of the Thesis

A thesis is a research document. It is a record of the research that the student conducted while completing a graduate degree. Given the research and education mandate of Canada's publicly funded universities, it is expected that the results of this research will be made publicly available.

For those theses that are submitted through the Scholarship@Western ETD Repository, the University requires that successful graduate theses be made available through the Western Library's Scholarship@Western program. The thesis will be published electronically at the conclusion of the degree process, and will be available globally via the World Wide Web.

At the same time, the University recognizes that the student is the author of the thesis, and retains copyright and control interests in the material.

Note: Students should be conscious of the implications of electronic publication in the digital context: material is accessible to any interested party, academic and non-academic. The record should also be understood to be permanently available – once published electronically, it can be withdrawn from Scholarship@Western, but digital copies will inevitably persist. Students engaging in thesis preparation and research should be mindful of electronic publication and availability as an endpoint of their research. Supervisors, equally, have a responsibility to be acquainted with the implications of electronic publication, and advise their students accordingly.

In certain cases, a "delay of publication" may be appropriate. See section 1.5.3.2 above.

The candidate is asked to permit the release of the thesis to be used for research and also to agree to allow the non-exclusive right to reproduce or loan copies of the thesis in micro-form, paper, or electronic formats. The term "non-exclusive license" means that the author retains the copyright of the thesis and can seek other forms of publication.

Upon final approved submission, the work is published to the Scholarship@Western ETD repository, pending any requests for a delay of publication. This repository is publicly accessible, permitting free access to the work. The repository transmits regular reports via e-mail to the author on how often the work is accessed.

Chapter 2

Background Theory

Forward problem and inverse problem. INFO for intro here.

2.1 Elastic Media

Aki and Richards (1980) note that "a medium is said to be elastic if it possess a natural state (in which strains and stresses are zero) to which it will revert when applied forces are removed." It is important to note that we are assuming elasticity–that is that rocks that deform will come back to their original shape. For a seismic wave, the elastic assumption is reasonable for propagation through rocks, provided we are not close to the seismic source (where permanent deformations may occur). Consider, for example, a dynamite source that explodes about 30 feet below the surface. Some of the rock will permanently deform around the explosion, and will not obey the laws of elasticity.

The seismic wave equation for propagation in elastic solids can be derived from Hooke's law and Newton's second law of motion. We must begin by understanding the concepts of stress and strain in order to properly relate them using Hooke's law. We will consider both stress and strain in three dimensions (3D), and then reduce to two dimensions (2D). *Stress can be defined as force per unit area*, and *strain can be defined as extension per unit length*. Seismic waves propagate through rocks and as they propagate there is a force that acts upon the rocks (stress). This stress that acts on the rocks induces slight deformations (strain). The derivations that follow are in large parts with help from Yilmaz (2001), Aki and Richards (1980), Thomsen (1986), and Tsvankin (2012).

2.1.1 Stress

Let us first draw an infinitesimally small cube with dimensions δx, δy, and δz (Figure 2.1).

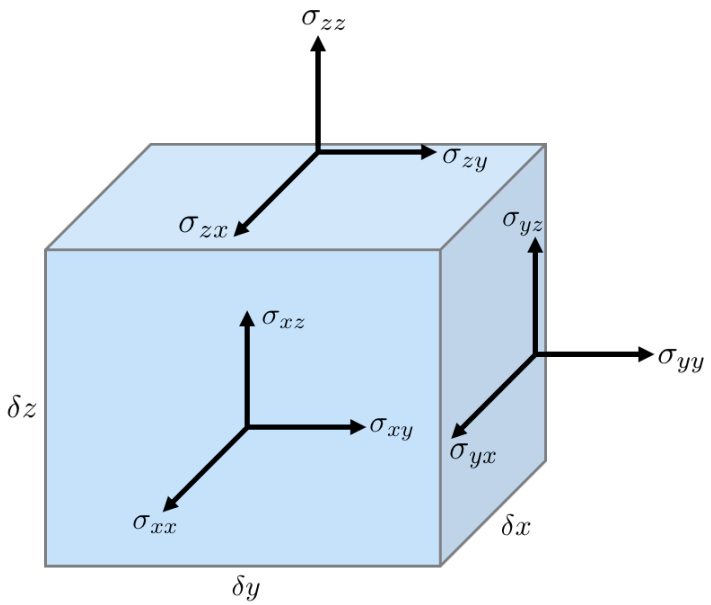

Figure 2.1: Infinitesimally small cube of dimensions $\delta x \times \delta y \times \delta z$, with all nine stress components labelled (adapted from Yilmaz (2001)).

Stress can act on any one of the surfaces of this cube. Let us consider that stress acts on the surface of $\delta y \times \delta z$, and define σ_{xx} to be the stress acting on that surface in the normal direction. A positive σ_{xx} is said to be *tensional stress* and a negative σ_{xx} is said to be *compressional stress.* There are also stresses that act parallel to the surface, defined as σ_{xy} and σ_{xz}. The first subscript indicates the normal direction to the surface, while the second indicates the stress component direction. For the surfaces $\delta x \times \delta y$ and $\delta x \times \delta z$, we may construct similar definitions. We can represent each of these stresses in what is commonly known as the Cauchy stress tensor matrix:

$$\sigma_{ij} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{pmatrix}. \tag{2.1}$$

The diagonal elements, σ_{xx}, σ_{yy}, and σ_{zz}, are known as the *normal stress components*, while the off-diagonals, σ_{yx}, σ_{xy}, σ_{zx}, σ_{xz}, σ_{zy}, and σ_{yz}, are known as the *shear stress components*. If the dimensions of the cube are made infinitesimally small, then the sum of the surface forces about any axis will become 0:

$$\begin{aligned} \sigma_{xy} - \sigma_{yx} &= 0 \\ \sigma_{xz} - \sigma_{zx} &= 0 \\ \sigma_{yz} - \sigma_{zy} &= 0, \end{aligned}$$

or $\sigma_{xy} = \sigma_{yx}$, $\sigma_{xz} = \sigma_{zx}$, and $\sigma_{zy} = \sigma_{yz}$, making the stress tensor matrix symmetrical. For an elaborated discussion on reasons for symmetry of the stress tensor, refer to Aki and Richards (1980).

2.1.2 Strain

Strain can be defined as extension per unit length, and is thus a dimensionless quality. Let us consider two points, A and B, within a solid body being subject to stress. They are deformed and become new points A' and B'. The displacement from from old to new point locations can be expressed by

$$\begin{pmatrix} \delta u_x \\ \delta u_y \\ \delta u_z \end{pmatrix} = \begin{pmatrix} \partial u_x/\partial x & \partial u_x/\partial y & \partial u_x/\partial z \\ \partial u_y/\partial x & \partial u_y/\partial y & \partial u_y/\partial z \\ \partial u_z/\partial x & \partial u_z/\partial y & \partial u_z/\partial z \end{pmatrix} \begin{pmatrix} \delta x \\ \delta y \\ \delta z \end{pmatrix}, \tag{2.2}$$

Expanding the first row would result in a displacement of u:

$$\delta u_x = \frac{\partial u_x}{\partial x}\delta x + \frac{\partial u_x}{\partial y}\delta y + \frac{\partial u_x}{\partial z}\delta z. \tag{2.3}$$

The displacements in (2.2) can be written (Aki and Richards, 1980) in a more compact notation:

$$\delta u_i = \frac{\partial u_i}{\partial x_j}\delta x_j. \tag{2.4}$$

When the displacement between two points becomes very small, we can set the derivative of u_i equal to the diplacement. Rearranging equation (2.4), we see that we have obtained this result:

$$\frac{\partial u_i}{\partial x_j} = \frac{\delta u_i}{\delta x_j}.$$

The displacement equation equation (2.2) is for some general displacement, but a solid body (or rock) can be deformed in various ways. Similar to the stress tensor matrix, the strain tensor matrix can be written as

$$e_{kl} = \begin{pmatrix} e_{xx} & e_{xy} & e_{xz} \\ e_{yx} & e_{yy} & e_{yz} \\ e_{zx} & e_{zy} & e_{zz} \end{pmatrix}, \tag{2.5}$$

where the diagonal elements represent the *normal strain* components, and the off-diagonal components represent *shear strain* components. The normal strain components are defined in terms of particle displacement u_i as

$$e_{xx} = \frac{\partial u_x}{\partial x}, \tag{2.6}$$

$$e_{yy} = \frac{\partial u_y}{\partial y}, \tag{2.7}$$

and

$$e_{zz} = \frac{\partial u_z}{\partial z}. \tag{2.8}$$

A positive normal strain is an *extension*, while a negative normal strain is a *contraction*. We now consider only three more types of strain that are sufficiently able to approximate every kind of strain to an infinitesimally small cube: linear stretching, shearing, rotation, and a combination of both shearing and rotation.

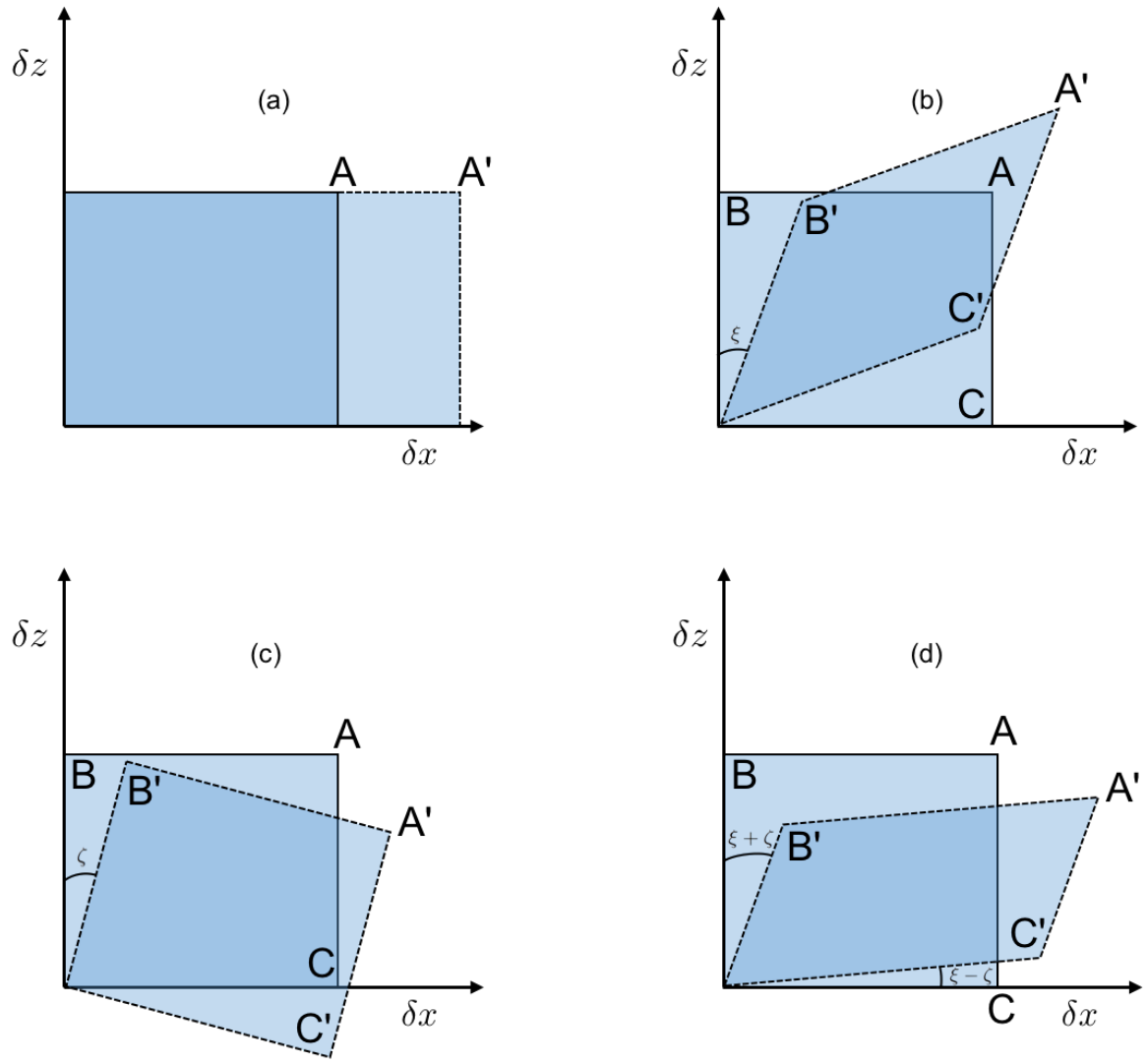

Figure 2.2: Stress causes slight deformations (strain), depicted here on the $\delta x \times \delta z$ plane: (a) linear 'stretching'; (b) shearing only; (c) rotation only; (d) combined shearing and rotation. Adapted from Yilmaz (2001).

We are thus assuming that whatever shape in deformation occurs, the shapes presented here are good estimates for *any* deformation. Because the cube is assumed to be infinitesimally small, this is a reasonable assumption.

In order to examine angular deformations, let us first define define angular deformations ξ and ζ in the $\delta x \times \delta z$ plane:

$$\xi = e_{xz} = e_{zx} \tag{2.9}$$

and

$$\zeta = \theta_{xz} = -\theta_{zx} \tag{2.10}$$

(Figure 2.2 (b) and (c)). As the volume becomes infinitesimally small, the angular deformations can be defined in terms of displacement, such that

$$\xi - \zeta = \frac{\partial u_z}{\partial x} \tag{2.11}$$

and

$$\xi + \zeta = \frac{\partial u_x}{\partial z} \tag{2.12}$$

(Figure 2.2 (d)). Adding equations equation (2.11) and equation (2.12), we get

$$2\xi = \frac{\partial u_z}{\partial x} + \frac{\partial u_x}{\partial z}.$$

Thus,

$$\xi = e_{xz} = \frac{1}{2}\left(\frac{\partial u_z}{\partial x} + \frac{\partial u_x}{\partial z}\right) = e_{zx}. \tag{2.13}$$

By the same analysis in the $\delta x \times \delta y$ plane, we get

$$e_{xy} = \frac{1}{2}\left(\frac{\partial u_y}{\partial x} + \frac{\partial u_x}{\partial y}\right) = e_{yx} \tag{2.14}$$

and in the $\delta y \times \delta z$ plane,

$$e_{yz} = \frac{1}{2}\left(\frac{\partial u_z}{\partial y} + \frac{\partial u_y}{\partial z}\right) = e_{zy}. \tag{2.15}$$

The relationship between strain, e_{ij}, and particle displacement, u, can be written compactly for all cases as

$$e_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial j} + \frac{\partial u_j}{\partial i}\right), i, j = x, y, z.$$

Equations (2.13), (2.14), and (2.15) represent the shear strain components from equation (2.5). The rotational deformation component must now be considered. Beginning in the $\delta x \times \delta z$ plane, and subtracting equations equation (2.11) and equation (2.12), we get

$$-2\zeta = \frac{\partial u_z}{\partial x} - \frac{\partial u_x}{\partial z}.$$

Thus,

$$\zeta = \theta_{xz} = -\frac{1}{2}\left(\frac{\partial u_z}{\partial x} - \frac{\partial u_x}{\partial z}\right) = -\theta_{zx} \tag{2.16}$$

In the $\delta x \times \delta y$ plane,

$$\theta_{xy} = -\frac{1}{2}\left(\frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y}\right) = -\theta_{yx} \tag{2.17}$$

and in the $\delta y \times \delta z$ plane,

$$\theta_{yz} = -\frac{1}{2}\left(\frac{\partial u_z}{\partial y} - \frac{\partial u_y}{\partial z}\right) = -\theta_{zy}. \tag{2.18}$$

The displacement tensor in equation equation (2.2) can now be written in an expanded form such that

$$\begin{pmatrix} \delta u_x \\ \delta u_y \\ \delta u_z \end{pmatrix} = \frac{1}{2}\left[\begin{pmatrix} \partial u_x/\partial x + \partial u_x/\partial x & \partial u_y/\partial x + \partial u_x/\partial y & \partial u_z/\partial x + \partial u_x/\partial z \\ \partial u_x/\partial y + \partial u_y/\partial x & \partial u_y/\partial y + \partial u_y/\partial y & \partial u_z/\partial y + \partial u_y/\partial z \\ \partial u_x/\partial z + \partial u_z/\partial x & \partial u_y/\partial z + \partial u_z/\partial y & \partial u_z/\partial z + \partial u_z/\partial z \end{pmatrix} - \frac{1}{2}\begin{pmatrix} 0 & \partial u_y/\partial x - \partial u_x/\partial y & \partial u_z/\partial x - \partial u_x/\partial z \\ \partial u_x/\partial y - \partial u_y/\partial x & 0 & \partial u_z/\partial y - \partial u_y/\partial z \\ \partial u_x/\partial z - \partial u_z/\partial x & \partial u_y/\partial z - \partial u_z/\partial y & 0 \end{pmatrix}\right]\begin{pmatrix} \delta x \\ \delta y \\ \delta z \end{pmatrix},$$

reducing to

$$\begin{pmatrix} \delta u_x \\ \delta u_y \\ \delta u_z \end{pmatrix} = \left[\begin{pmatrix} e_{xx} & e_{xy} & e_{xz} \\ e_{yx} & e_{yy} & e_{yz} \\ e_{zx} & e_{zy} & e_{zz} \end{pmatrix} + \begin{pmatrix} 0 & \theta_{xy} & \theta_{xz} \\ \theta_{yx} & 0 & \theta_{yz} \\ \theta_{zx} & \theta_{zy} & 0 \end{pmatrix}\right]\begin{pmatrix} \delta x \\ \delta y \\ \delta z \end{pmatrix}. \tag{2.19}$$

2.1.3 Hooke's Law and Elastic Tensor

Next, we need to establish a relationship between the Cauchy stress tensor (2.1) and the strain tensor (2.5). They can be related by what is often called the generalized Hooke's law, expressed by

$$\sigma_{ij} = c_{ijkl} e_{kl}, \tag{2.20}$$

where σ_{ij} is the stress tensor, c_{ijkl} is the elastic (or 'stiffness') tensor, and e_{kl} is the strain tensor. The $3 \times 3 \times 3 \times 3$ elastic tensor (c_{ijkl}) has several symetries that immediately reduce the number of elastic constants from 81 to 21. The elastic tensor $\mathbf{c}$ of rank four with 81 constants can be written (Heilbig, 1994) in three-dimensional space as

$$\mathbf{c} = \begin{pmatrix} \begin{pmatrix} c_{1111} & c_{1112} & c_{1113} \\ c_{1112} & c_{1122} & c_{1123} \\ c_{1113} & c_{1123} & c_{1133} \end{pmatrix} & \begin{pmatrix} c_{1211} & c_{1212} & c_{1213} \\ c_{1212} & c_{1222} & c_{1223} \\ c_{1213} & c_{1223} & c_{1233} \end{pmatrix} & \begin{pmatrix} c_{1311} & c_{1312} & c_{1313} \\ c_{1312} & c_{1322} & c_{1323} \\ c_{1313} & c_{1323} & c_{1333} \end{pmatrix} \\ \begin{pmatrix} c_{1211} & c_{1212} & c_{1213} \\ c_{1212} & c_{1222} & c_{1223} \\ c_{1213} & c_{1223} & c_{1233} \end{pmatrix} & \begin{pmatrix} c_{2211} & c_{2212} & c_{2213} \\ c_{2212} & c_{2222} & c_{2223} \\ c_{2213} & c_{2223} & c_{2233} \end{pmatrix} & \begin{pmatrix} c_{2311} & c_{2312} & c_{2313} \\ c_{2312} & c_{2322} & c_{2323} \\ c_{2313} & c_{2323} & c_{2333} \end{pmatrix} \\ \begin{pmatrix} c_{1311} & c_{1312} & c_{1313} \\ c_{1312} & c_{1322} & c_{1323} \\ c_{1313} & c_{1323} & c_{1333} \end{pmatrix} & \begin{pmatrix} c_{2311} & c_{2312} & c_{2313} \\ c_{2312} & c_{2322} & c_{2323} \\ c_{2313} & c_{2323} & c_{2333} \end{pmatrix} & \begin{pmatrix} c_{3311} & c_{3312} & c_{3313} \\ c_{3312} & c_{3322} & c_{3323} \\ c_{3313} & c_{3323} & c_{3333} \end{pmatrix} \end{pmatrix}. \tag{2.21}$$

The symmetry of stress ($\sigma_{ij} = \sigma_{ji}$) and the symmetry of strain ($e_{kl} = e_{lk}$) result in equalities

$$c_{ijkl} = c_{jikl} \tag{2.22}$$

and

$$c_{ijkl} = c_{ijlk}. \tag{2.23}$$

It is also true from a thermodynamic standpoint that

$$c_{ijkl} = c_{klij}. \tag{2.24}$$

Aki and Richards (1980) offer a proof beginning from the first law of thermodynamics to establish (2.24). Following from equations (2.22), (2.23), and (2.24), the stiffness tensor can be written in the form of a 6×6 matrix, with only 21 independent constants. To simplify notation, the "Voigt

recipe" for indices can be used as follows:

$$\begin{array}{cccccc} 11 & 22 & 33 & 23{=}32 & 13{=}31 & 12{=}21 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 1 & 2 & 3 & 4 & 5 & 6 \end{array} .$$

When anisotropic symmetries are increased, the number of independent elastic constants decreases. The new elastic tensor is of rank two with 21 independent constants and can be written in what is called triclinic form, with no symmetry planes (Tsvankin, 2012):

$$\mathbf{c}^{(\mathrm{TRC})} = \begin{pmatrix} c_{11} & c_{12} & c_{13} & c_{14} & c_{15} & c_{16} \\ c_{12} & c_{22} & c_{23} & c_{24} & c_{25} & c_{26} \\ c_{13} & c_{23} & c_{33} & c_{34} & c_{35} & c_{36} \\ c_{14} & c_{24} & c_{34} & c_{44} & c_{45} & c_{46} \\ c_{15} & c_{25} & c_{35} & c_{45} & c_{55} & c_{56} \\ c_{16} & c_{26} & c_{36} & c_{46} & c_{56} & c_{66} \end{pmatrix}. \tag{2.25}$$

Equation (2.25) is known as the most general form of anisotropy for the elastic tensor. Notice that the matrix is completely symmetric, reducing the number of independent constants from what would be 36 in a 6×6 matrix to only 21. The generalized Hooke's law given by (2.20) can be written in expanded form relating stress to strain such that

$$\begin{pmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{xy} \\ \sigma_{xz} \\ \sigma_{yz} \end{pmatrix} = \begin{pmatrix} c_{11} & c_{12} & c_{13} & c_{14} & c_{15} & c_{16} \\ c_{12} & c_{22} & c_{23} & c_{24} & c_{25} & c_{26} \\ c_{13} & c_{23} & c_{33} & c_{34} & c_{35} & c_{36} \\ c_{14} & c_{24} & c_{34} & c_{44} & c_{45} & c_{46} \\ c_{15} & c_{25} & c_{35} & c_{45} & c_{55} & c_{56} \\ c_{16} & c_{26} & c_{36} & c_{46} & c_{56} & c_{66} \end{pmatrix} \begin{pmatrix} e_{xx} \\ e_{yy} \\ e_{zz} \\ e_{xy} \\ e_{xz} \\ e_{yz} \end{pmatrix}. \tag{2.26}$$

In seismology, it is often necessary to simplify the stiffness tensor matrix for geological reasons–that is–to reduce the number of parameters in (2.25) for robust solutions. An elastic medium can be simplified by taking certain axes to be symmetric. In exploration seismology, the tensors that follow are some of the most common simplifications of equation (2.25), according to specific symmetry assumptions Tsvankin (2012). The tensors are named according to symmetry axes:

1. monoclinic media, with ?

$$\mathbf{c}^{(\mathrm{MNC})} = \begin{pmatrix} c_{11} & c_{12} & c_{13} & & & c_{16} \\ c_{12} & c_{22} & c_{23} & & & c_{26} \\ c_{13} & c_{23} & c_{33} & & & c_{36} \\ & & & c_{44} & c_{45} & \\ & & & c_{45} & c_{55} & \\ c_{16} & c_{26} & c_{36} & & & c_{66} \end{pmatrix}; \tag{2.27}$$

2. orthorhombic media, with three mutually orthogonal planes of reflection symmetry:

$$\mathbf{c}^{(\mathrm{ORT})} = \begin{pmatrix} c_{11} & c_{12} & c_{13} & & & \\ c_{12} & c_{22} & c_{23} & & & \\ c_{13} & c_{23} & c_{33} & & & \\ & & & c_{44} & & \\ & & & & c_{55} & \\ & & & & & c_{66} \end{pmatrix}; \tag{2.28}$$

3. transversely isotropic media, with a ?, including vertical transverse isotropy (VTI) ***important to talk about flat shale layers most common in exploration seismology VTI is most common (as in Firestone dataset!)***

$$\mathbf{c}^{(\mathrm{VTI})} = \begin{pmatrix} c_{11} & c_{11}-2c_{66} & c_{13} & & & \\ c_{11}-2c_{66} & c_{22} & c_{13} & & & \\ c_{13} & c_{13} & c_{33} & & & \\ & & & c_{44} & & \\ & & & & c_{44} & \\ & & & & & c_{66} \end{pmatrix}, \tag{2.29}$$

tilted transverse isotropy (TTI), and horizontal transverse isotropy (HTI):

$$\mathbf{c}^{(\mathrm{HTI})} = \begin{pmatrix} c_{11} & c_{13} & c_{13} & & & \\ c_{13} & c_{22} & c_{33}-2c_{44} & & & \\ c_{13} & c_{33}-2c_{44} & c_{33} & & & \\ & & & c_{44} & & \\ & & & & c_{55} & \\ & & & & & c_{55} \end{pmatrix}; \tag{2.30}$$

and

4. isotropic media, with ??:

Table 2.1: Number of independent parameters for the elastic tensors (2.25), (2.27), (2.28), (2.29), (2.30), and (2.31).

Symmetry	**Elastic Parameters**
triclinic	21
monoclinic	13
orthorhombic	9
transversely isotropic (VTI, TTI, HTI)	5
isotropic	2

$$\mathbf{c}^{(\mathrm{ISO})} = \begin{pmatrix} c_{33} & c_{33}-2c_{44} & c_{33}-2c_{44} & & & & \\ c_{33}-2c_{44} & c_{33} & c_{33}-2c_{44} & & & & \\ c_{33}-2c_{44} & c_{33}-2c_{44} & c_{33} & & & & \\ & & & c_{44} & & & \\ & & & & c_{44} & & \\ & & & & & c_{44} & \end{pmatrix}, \tag{2.31}$$

also commonly written as

$$\mathbf{c}^{(\mathrm{ISO})} = \begin{pmatrix} \lambda+2\mu & \lambda & \lambda & & & & \\ \lambda & \lambda+2\mu & \lambda & & & & \\ \lambda & \lambda & \lambda+2\mu & & & & \\ & & & \mu & & & \\ & & & & \mu & & \\ & & & & & \mu & \end{pmatrix}, \tag{2.32}$$

where λ and μ are referred to as the Lamé parameters, or elastic moduli. These parameters constitute what is known as the bulk modulus for homogeneous isotropic media,

$$k = \lambda + \frac{2}{3}\mu. \tag{2.33}$$

It is helpful at this point to summarize the number of parameters for each elastic tensor[1]:

The elastic tensor in various media described by (2.25), (2.27), (2.28), (2.29), (2.30), and (2.31) have been defined by crystal lattice structures in solid state physics. For a comprehensive review of

[1]Helbig (1994) uses slightly different numbers in his table for triclinic and monoclinic media, as there are further simplifications that can be made to reduce these number of parameters; these simplifications can be explored further in Helbig (1994) and Tsvankin (2012).

the theory of elastic waves in crystals, refer to Fedorov (1968), Nye (1957), Helbig and Schoenberg (1987), and Helbig (1994).

2.1.4 Equations of Motion and Wave Equations

Newton's equations of motion can be written in the frequency domain as

$$-\omega^2 \rho u_x = \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z} + f_x, \tag{2.34}$$

$$-\omega^2 \rho u_y = \frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{yz}}{\partial z} + f_y, \tag{2.35}$$

and

$$-\omega^2 \rho u_z = \frac{\partial \sigma_{xz}}{\partial x} + \frac{\partial \sigma_{yz}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} + f_z, \tag{2.36}$$

where ω is the angular frequency; ρ is the density; u_i $(i = x, y, z)$ are the particle displacements; σ_{ij} $(i, j = x, y, z)$ are the stress components; and f_i $(i = x, y, z)$ are the source terms. By substituting what we know from Hooke's law into the equations of motion, we can obtain the wave equation.

2.1.4.1 3D VTI Elastic Wave Equation

If we expand Hooke's law for VTI media,

$$\begin{pmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{xy} \\ \sigma_{xz} \\ \sigma_{yz} \end{pmatrix} = \begin{pmatrix} c_{11} & c_{11}-2c_{66} & c_{13} & & & \\ c_{11}-2c_{66} & c_{22} & c_{13} & & & \\ c_{13} & c_{13} & c_{33} & & & \\ & & & c_{44} & & \\ & & & & c_{44} & \\ & & & & & c_{66} \end{pmatrix} \begin{pmatrix} e_{xx} \\ e_{yy} \\ e_{zz} \\ e_{xy} \\ e_{xz} \\ e_{yz} \end{pmatrix}, \tag{2.37}$$

we get the system of equations that relate stress to strain:

$$\begin{aligned}
\sigma_{xx} &= c_{11}e_{xx} + (c_{11} - 2c_{66})e_{yy} + c_{13}e_{zz},\\
\sigma_{yy} &= (c_{11} - 2c_{66})e_{xx} + c_{22}e_{yy} + c_{13}e_{zz},\\
\sigma_{zz} &= c_{13}e_{xx} + c_{13}e_{yy} + c_{33}e_{zz},\\
\sigma_{xy} &= c_{44}e_{xy},\\
\sigma_{xz} &= c_{44}e_{xz},\\
\sigma_{yz} &= c_{66}e_{yz},
\end{aligned}$$

so that our new equations of motion become

$$-\omega^2\rho u_x = \frac{\partial}{\partial x}\left[c_{11}e_{xx} + (c_{11} - 2c_{66})e_{yy} + c_{13}e_{zz}\right] + \frac{\partial}{\partial y}c_{44}e_{xy} + \frac{\partial}{\partial z}(c_{44}e_{xz}) + f_x, \tag{2.38}$$

$$-\omega^2\rho u_y = \frac{\partial}{\partial x}\left(c_{44}e_{xy}\right) + \frac{\partial}{\partial y}\left[(c_{11} - 2c_{66})e_{xx} + c_{22}e_{yy} + c_{13}e_{zz}\right] + \frac{\partial}{\partial z}\left(c_{66}e_{yz}\right) + f_y, \tag{2.39}$$

and

$$-\omega^2\rho u_z = \frac{\partial}{\partial x}\left(c_{44}e_{xz}\right) + \frac{\partial}{\partial y}\left(c_{66}e_{yz}\right) + \frac{\partial}{\partial z}\left(c_{13}e_{xx} + c_{13}e_{yy} + c_{33}e_{zz}\right) + f_z. \tag{2.40}$$

Then, if we substitute what we know about strain (e_{ij}) from (2.6), (2.7), (2.8), (2.13), (2.14), and (2.15), we then get the **elastic wave equations for inhomogeneous 3D VTI media**

$$\begin{aligned}
-\omega^2\rho u_x = \frac{\partial}{\partial x}\left[c_{11}\frac{\partial u_x}{\partial x} + (c_{11} - 2c_{66})\frac{\partial u_y}{\partial y} + c_{13}\frac{\partial u_z}{\partial z}\right] + \frac{\partial}{\partial y}\left[\frac{1}{2}c_{44}\left(\frac{\partial u_z}{\partial y} + \frac{\partial u_y}{\partial z}\right)\right] +\\
\frac{\partial}{\partial z}\left[\frac{1}{2}c_{44}\left(\frac{\partial u_z}{\partial x} + \frac{\partial u_x}{\partial z}\right)\right] + f_x,
\end{aligned} \tag{2.41}$$

$$\begin{aligned}
-\omega^2\rho u_y = \frac{\partial}{\partial x}\left[\frac{1}{2}c_{44}\left(\frac{\partial u_y}{\partial x} + \frac{\partial u_x}{\partial y}\right)\right] + \frac{\partial}{\partial y}\left[(c_{11} - 2c_{66})\frac{\partial u_x}{\partial x} + c_{22}\frac{\partial u_y}{\partial y} + c_{13}\frac{\partial u_z}{\partial z}\right] +\\
\frac{\partial}{\partial z}\left[\frac{1}{2}c_{66}\left(\frac{\partial u_z}{\partial y} + \frac{\partial u_y}{\partial z}\right)\right] + f_y,
\end{aligned} \tag{2.42}$$

and

$$-\omega^2\rho u_z = \frac{\partial}{\partial x}\left[\frac{1}{2}c_{44}\left(\frac{\partial u_z}{\partial x}+\frac{\partial u_x}{\partial z}\right)\right]+\frac{\partial}{\partial y}\left[\frac{1}{2}c_{66}\left(\frac{\partial u_z}{\partial y}+\frac{\partial u_y}{\partial z}\right)\right]+ \frac{\partial}{\partial z}\left[c_{13}\frac{\partial u_x}{\partial x}+c_{13}\frac{\partial u_y}{\partial y}+c_{33}\frac{\partial u_z}{\partial z}\right]+f_z. \tag{2.43}$$

Thomsen (1986) defined three anisotropy parameters that have become so widely used that they are now commonly referred to as the 'Thomsen parameters.' They are defined as follows for VTI media:

$$\varepsilon \equiv \frac{c_{11}-c_{33}}{2c_{33}}, \tag{2.44}$$

$$\gamma \equiv \frac{c_{66}-c_{44}}{2c_{44}}, \tag{2.45}$$

and

$$\delta \equiv \frac{(c_{13}+c_{44})^2-(c_{33}-c_{44})^2}{2c_{33}(c_{33}-c_{44})}. \tag{2.46}$$

The vertical sound speeds of P- and S-waves are

$$\alpha_0 = \sqrt{\frac{c_{33}}{\rho}} \tag{2.47}$$

and

$$\beta_0 = \sqrt{\frac{c_{44}}{\rho}}, \tag{2.48}$$

respectively. Thomsen (1986) determined that most rocks are only 'weakly' anisotropic, which typically exhibit anisotropy values ε, γ, and $\delta < 0.2$. For weak anisotropy, he defined the phase velocities as

$$v_P(\theta) = \alpha_0(1+\delta\sin^2\theta\cos^2\theta+\varepsilon\sin^4\theta), \tag{2.49}$$

$$v_{SV}(\theta) = \beta_0 \left[1 + \frac{\alpha_0^2}{\beta_0^2}(\varepsilon - \delta)\, \sin^2\theta \cos^2\theta \right], \tag{2.50}$$

and

$$v_{SH}(\theta) = \beta_0(1 + \gamma\, \sin^2\theta), \tag{2.51}$$

where $v_P(\theta)$ is the P-wave velocity, $v_{SV}(\theta)$ is the S-wave velocity in the vertical direction ('SV-wave'), and $v_{SH}(\theta)$ is the S-wave velocity in the horizontal direction ('Shear-wave').

2.1.4.2 3D Isotropic Elastic Wave Equation

The isotropic wave equation has only two parameters, μ and λ. In a similar fashion to the VTI expansion, we begin with Hooke's law for isotropic media,

$$\begin{pmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{xy} \\ \sigma_{xz} \\ \sigma_{yz} \end{pmatrix} = \begin{pmatrix} c_{33} & c_{33}-2c_{44} & c_{33}-2c_{44} & & & \\ c_{33}-2c_{44} & c_{33} & c_{33}-2c_{44} & & & \\ c_{33}-2c_{44} & c_{33}-2c_{44} & c_{33} & & & \\ & & & c_{44} & & \\ & & & & c_{44} & \\ & & & & & c_{44} \end{pmatrix} \begin{pmatrix} e_{xx} \\ e_{yy} \\ e_{zz} \\ e_{xy} \\ e_{xz} \\ e_{yz} \end{pmatrix} \tag{2.52}$$

$$= \begin{pmatrix} \lambda+2\mu & \lambda & \lambda & & & \\ \lambda & \lambda+2\mu & \lambda & & & \\ \lambda & \lambda & \lambda+2\mu & & & \\ & & & \mu & & \\ & & & & \mu & \\ & & & & & \mu \end{pmatrix} \begin{pmatrix} e_{xx} \\ e_{yy} \\ e_{zz} \\ e_{xy} \\ e_{xz} \\ e_{yz} \end{pmatrix}, \tag{2.53}$$

and relate it to the equations of motion (2.34), (2.35), and (2.36). The **elastic wave equations**

for inhomogeneous 3D isotropic media then expand to

$$-\omega^2\rho u_x = \frac{\partial}{\partial x}\left[\lambda\left(\frac{\partial u_x}{\partial x}+\frac{\partial u_y}{\partial y}+\frac{\partial u_z}{\partial z}\right)+2\mu\frac{\partial u_x}{\partial x}\right]+\frac{\partial}{\partial y}\left[\frac{1}{2}\mu\left(\frac{\partial u_z}{\partial y}+\frac{\partial u_y}{\partial z}\right)\right]+\frac{\partial}{\partial z}\left[\frac{1}{2}\mu\left(\frac{\partial u_z}{\partial x}+\frac{\partial u_x}{\partial z}\right)\right]+f_x, \tag{2.54}$$

$$-\omega^2\rho u_y = \frac{\partial}{\partial x}\left[\frac{1}{2}\mu\left(\frac{\partial u_y}{\partial x}+\frac{\partial u_x}{\partial y}\right)\right]+\frac{\partial}{\partial y}\left[\lambda\left(\frac{\partial u_x}{\partial x}+\frac{\partial u_y}{\partial y}+\frac{\partial u_z}{\partial z}\right)+2\mu\frac{\partial u_y}{\partial y}\right]+\frac{\partial}{\partial z}\left[\frac{1}{2}\mu\left(\frac{\partial u_z}{\partial y}+\frac{\partial u_y}{\partial z}\right)\right]+f_y, \tag{2.55}$$

and

$$-\omega^2\rho u_z = \frac{\partial}{\partial x}\left[\frac{1}{2}\mu\left(\frac{\partial u_z}{\partial x}+\frac{\partial u_x}{\partial z}\right)\right]+\frac{\partial}{\partial y}\left[\frac{1}{2}\mu\left(\frac{\partial u_z}{\partial y}+\frac{\partial u_y}{\partial z}\right)\right]+\frac{\partial}{\partial z}\left[\lambda\left(\frac{\partial u_x}{\partial x}+\frac{\partial u_y}{\partial y}+\frac{\partial u_z}{\partial z}\right)+2\mu\frac{\partial u_z}{\partial z}\right]+f_z. \tag{2.56}$$

Thomsen's parameters for isotropic media are defined as

$$\varepsilon \equiv \frac{c_{11}-c_{33}}{2c_{33}} \equiv 0, \tag{2.57}$$

and therefore

$$c_{11} = c_{33};$$

$$\gamma \equiv \frac{c_{66}-c_{44}}{2c_{44}} \equiv 0, \tag{2.58}$$

and therefore

$$c_{44} = c_{66};$$

$$\delta \equiv \frac{(c_{13}+c_{44})^2-(c_{33}-c_{44})^2}{2c_{33}(c_{33}-c_{44})} \equiv 0, \tag{2.59}$$

and therefore

$$c_{13} = c_{33} - 2c_{44};$$

$$\alpha_0 = \sqrt{\frac{c_{33}}{\rho}} = \sqrt{\frac{\lambda + 2\mu}{\rho}}; \tag{2.60}$$

and

$$\beta_0 = \sqrt{\frac{c_{44}}{\rho}} = \sqrt{\frac{\mu}{\rho}}. \tag{2.61}$$

The velocities in isotropic media simplify to

$$v_P = \alpha_0 \tag{2.62}$$

$$v_{SV} = \beta_0 \tag{2.63}$$

and

$$v_{SH} = \beta_0, \tag{2.64}$$

now no longer dependent on a phase angle θ. Notice also that in isotropic media $v_{SV} = v_{SH}$. There is no division of S-waves into SV-waves and SH-waves, because the phase angle is no longer a factor.

2.1.4.3 Elastic Wave Equations in 2D

In seismic exploration, most datasets have historically been recorded in only two dimensions, and therefore the forward modeling code and inversion code required also need to be in two dimensions. Though we are now beginning to record more seismic data in 3D, acquisition in 3D is still expensive. The computational costs for dealing with 3D data are also very expensive, and for the purposes of the work of this thesis, we remain in 2D. It is important, then, to be able to accurately describe the wave equations in 2D. Working from (2.41), (2.42), and (2.43), when reducing to 2D,

we now have the displacement of y as 0, or

$$\frac{\partial}{\partial y}(X) \equiv 0,$$

where X is any arbitrary variable, then the **elastic wave equations for inhomogeneous 2D VTI** media then become

$$-\omega^2 \rho u_x = \frac{\partial}{\partial x}\left[c_{11}\frac{\partial u_x}{\partial x} + c_{13}\frac{\partial u_z}{\partial z}\right] + \frac{\partial}{\partial z}\left[\frac{1}{2}c_{44}\left(\frac{\partial u_z}{\partial x} + \frac{\partial u_x}{\partial z}\right)\right] + f_x, \tag{2.65}$$

$$-\omega^2 \rho u_y = \frac{\partial}{\partial x}\left[\frac{1}{2}c_{44}\left(\frac{\partial u_y}{\partial x}\right)\right] + \frac{\partial}{\partial z}\left[\frac{1}{2}c_{66}\left(\frac{\partial u_y}{\partial z}\right)\right] + f_y, \tag{2.66}$$

and

$$-\omega^2 \rho u_z = \frac{\partial}{\partial x}\left[\frac{1}{2}c_{44}\left(\frac{\partial u_z}{\partial x} + \frac{\partial u_x}{\partial z}\right)\right] + \frac{\partial}{\partial z}\left[c_{13}\frac{\partial u_x}{\partial x} + c_{33}\frac{\partial u_z}{\partial z}\right] + f_z. \tag{2.67}$$

For VTI media, the elastic tensors can now be described by a 3×3 matrix, with only 4 independent elastic parameters,

$$\mathbf{c}^{(\text{VTI-2D})} = \begin{pmatrix} c_{11} & c_{13} & \\ c_{13} & c_{33} & \\ & & c_{44} \end{pmatrix}. \tag{2.68}$$

The **elastic wave equations for inhomogeneous 2D isotropic** media are also simplified, and become

$$-\omega^2 \rho u_x = \frac{\partial}{\partial x}\left[\lambda\left(\frac{\partial u_x}{\partial x} + \frac{\partial u_z}{\partial z}\right) + 2\mu\frac{\partial u_x}{\partial x}\right] + \frac{\partial}{\partial z}\left[\frac{1}{2}\mu\left(\frac{\partial u_z}{\partial x} + \frac{\partial u_x}{\partial z}\right)\right] + f_x, \tag{2.69}$$

$$-\omega^2 \rho u_y = \frac{\partial}{\partial x}\left[\frac{1}{2}\mu\left(\frac{\partial u_y}{\partial x}\right)\right] + \frac{\partial}{\partial z}\left[\frac{1}{2}\mu\left(\frac{\partial u_y}{\partial z}\right)\right] + f_y \tag{2.70}$$

and

$$-\omega^2 \rho u_z = \frac{\partial}{\partial x}\left[\frac{1}{2}\mu\left(\frac{\partial u_z}{\partial x} + \frac{\partial u_x}{\partial z}\right)\right] + \frac{\partial}{\partial z}\left[\lambda\left(\frac{\partial u_x}{\partial x} + \frac{\partial u_z}{\partial z}\right) + 2\mu\frac{\partial u_z}{\partial z}\right] + f_z. \tag{2.71}$$

The associated 3×3 elastic tensor can be defined by

$$\mathbf{c}^{(\text{ISO-2D})} = \begin{pmatrix} c_{33} & c_{33}-2c_{44} & \\ c_{33}-2c_{44} & c_{33} & \\ & & c_{44} \end{pmatrix} = \begin{pmatrix} \lambda+2\mu & \lambda & \\ \lambda & \lambda+2\mu & \\ & & \mu \end{pmatrix}. \tag{2.72}$$

2.1.4.4 Acoustic Wave Equation in 3D

There is another common much more simplified wave equation that is used, called the acoustic wave equation. The acoustic assumptions are that 1) the normal stresses are all equal,

$$\sigma_{xx} = \sigma_{yy} = \sigma_{zz} = \sigma,$$

and 2) the shear stresses are all zero,

$$\sigma_{xy} = \sigma_{xz} = \sigma_{yz} = 0,$$

or

$$\mu = 0,$$

we get

$$-\omega^2\rho u_x = \frac{\partial}{\partial x}\left[\lambda\left(\frac{\partial u_x}{\partial x}+\frac{\partial u_y}{\partial y}+\frac{\partial u_z}{\partial z}\right)\right]+f_x$$

$$-\omega^2\rho u_y = \frac{\partial}{\partial y}\left[\lambda\left(\frac{\partial u_x}{\partial x}+\frac{\partial u_y}{\partial y}+\frac{\partial u_z}{\partial z}\right)\right]+f_y$$

and

$$-\omega^2\rho u_z = \frac{\partial}{\partial z}\left[\lambda\left(\frac{\partial u_x}{\partial x}+\frac{\partial u_y}{\partial y}+\frac{\partial u_z}{\partial z}\right)\right]+f_z. \tag{2.73}$$

The .....?

2.2 Forward Problem

We choose to work in the frequency domain, which allows computational advantages over the time-domain. Some of these advantages include being able to solve the wave equation for multiple sources at a minimal cost, use accurate attenuation modeling, and being able to use a frequency-selection strategy Pratt (1990). The forward problem can be discretized in 2D and represented in the frequency domain by a matrix formalism,

$$\mathbf{Su} = \mathbf{f} \text{ or } \mathbf{u} = \mathbf{S^{-1}f} \tag{2.74}$$

where **S** is the complex impedance or 'finite-difference' matrix, **u** is the wavefield (particle displacements), and **f** is the source vector. In forward modelling, we solve for **u**, a vector of created or 'synthetic' data. Equation (2.74) is only representational in that we do not actually in practice invert the impedance matrix, **S** Pratt et al. (1998) In our case, we use matrix factoring methods.

2.2.1 Finite-Difference Method

There are numerous methods that have been developed to solve the elastic wave equation for 2D VTI media. The one we use is...

[Some of the possibilities include those presented by these direct sources: Min et al. (2004); Pratt (1990); and Käser and Dumbser (2006)

2.3 Waveform Inversion

The main objective in the seismic inverse problem is to find velocity models close to the true velocity models of the subsurface by minimizing the objective function. The objective function we use in our case is the l_2 norm of the residual of the synthetic data and the observed data and can be written as

$$E = \frac{1}{2} \left[\mathbf{u} - \mathbf{d}\right]^T \left[\mathbf{u} - \mathbf{d}\right]^*, \tag{2.75}$$

where **u** is a vector of the synthetic data and **d** is a vector of the observed data, where the subscript T designates the matrix transpose, and * the complex conjugate.

[Continue by describing 'gradient inversion method' described very fully by Pratt et al. (1998), and very succinctly by Lee et al. (2010)

EXTRA MATERIAL

An efficient form of frequency-domain mode lling (FDM) was developed by Pratt et al. (1998), which is the basis for our work here.

(just in case to include?): ...

$$\begin{pmatrix} \sigma_{xx} \\ \sigma_{zz} \\ \sigma_{xz} \end{pmatrix} = \begin{pmatrix} c_{11} & c_{13} & c_{15} \\ c_{13} & c_{33} & c_{35} \\ c_{15} & c_{35} & c_{55} \end{pmatrix} \begin{pmatrix} e_{xx} \\ e_{zz} \\ e_{xz} \end{pmatrix}. \tag{2.76}$$

Chapter 3

Synthetic

Synthetic with acoustic and elastic inversions / forward modelling

Chapter 4

Firestone Field Data

Firestone with acoustic and elastic inversions / forward modelling

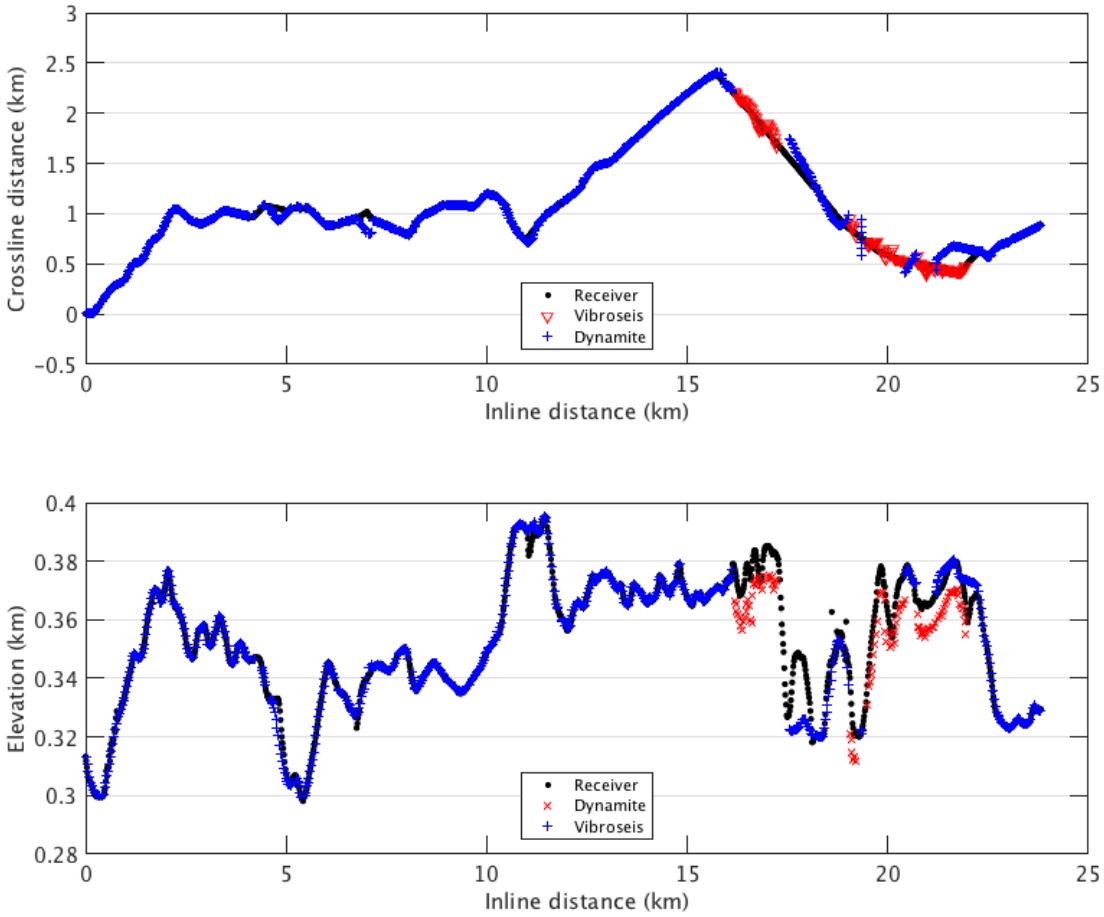

Figure 4.1: Firestone seismic line characteristics. In an actual figure and caption, you would definitely want to label them (a) and (b) and speak to each graph separately.

Chapter 5

Conclusions and Future Directions

Conclusions here

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Appendix A

Elastic Derivation

Type information here for Appendix A. Proof of theorems, etc.

Appendix B

Another Title

Type information here for Appendix B. Proof of theorems, etc.

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